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Vibrations Control of Two Degree of Freedom System Subject to External Force Using Passive and Time Delay Controllers

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ABSTRACT **: The main objective of this paper is a fine analytical and numerical study to consider the effects of time-delayed velocity feedback on a passive control a linear mass spring damper system subject to harmonic external force. The first-order approximate solution is applying the Multiple Scales Perturbation Technique (MSPT) to analyze the nonlinear behavior of this model. Studying the stability of the obtained numerical solution is investigated by using the phase plane methods and the frequency response equation in conjunction with the resonance cases ($\omega_1 = \Omega, \omega_1 = 2\omega_2$). We find that adding the absorber minimizes the amplitude of vibration in the steady state, so we can control the effective stiffness associated with the passive absorber. The problem has been solved by using the Runge-kutta method. Effects of different parameters on the system behavior are studied numerically by using the MATLAB program. Finally, a comparison of previously published work is done at the end of this work.**

KEYWORDS: ***Time-delay, Stability, Frequency response, Multiple Times Scale, Vibration Control, External Force, Passive Control.***

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I. INTRODUCTION

The vibration is the worst phenomenon that can be exposed to the structures or machines. An increasing rate of vibration causes the destruction and collapse of structures and damage machines, so we use different types of absorbers to control and reduce the damage caused by. One of the absorbers used to reduce the risk of vibration is time delay, where time delays are determined in any active or passive control system as a result of setting the system state. The existence of time delay imposes strict limitations on the control system. With delays in measurement, the absorber receives data via process behavior. Time delay is able to effect the stability of the system. The time delay on the control system may be a subject of researchers's interest.

Abdelhafez and Nassar. [1] Studied loop delays and they took into consideration at ion when a positive position feedback controller is used to control the vibrations of a forced and self-excited nonlinear beam. External excitation is a harmonic excitation caused by the support motion of the cantilever beam. Self-excitation is caused by fluid flow and modeled by a nonlinear damping with a negative linear part (Rayleigh's function). Yan et al. [2] verified the possibility of a vehicle suspension system under time delayed the optimal control. Showed how to effect of time delay on control stability of the action on the system. They used the mathematical simulation to verify the rightness of the stable interval obtained by differential equation theory for linear systems with constant coefficients and time delay.

Di Ferdinando and Pepe. [3] Their study involved examining the time delay of nonlinear systems and providing suitable conditions to enable the simulation of continuous-time dynamic output feedback controllers. The focus of the discussion was on stabilization issues in the sample-and-hold sense. As a specific case, they dealt with nonlinear systems with time delay.

Xu et al. [4] developed nonlinear saturation controller by using quadratic velocity coupling term with time delay instead of the original quadratic position coupling term in the controller. And they used controller development to control the high-amplitude vibration of a flexible, geometrically nonlinear beam-like structure when the primary resonance and the 1:2 internal resonances occur simultaneously. Coppola and Liu, [5] checked the characteristics of a unique isolation for active vibration controlled by a time-delayed position feedback technique. Where they found the system possesses a ratable time delay caused by the actuator dynamics. The study revealed several main differences between negative and positive position feedback. Then, they analyzed the stability of the delayed position feedback and studied two feedbacks, namely the delayed negative and delayed positive position feedbacks.

Sayed et al [6] found analytical and numerical study to investigate the vibration and stability of the Van der Pol equation subjected to external and parametric excitation forces via feedback control. The stability of the system is investigated applying Lyapunov's first method. The stability of the system is investigated applying Lyapunov's first method. Shin et al [7]. achieved control of the active vibrations in the clamps using the control unit in the positive feedback of the position with the torque sensor pair operator.

Ouakad et al. [8] improved the behavior of a micro beam by using a nonlinear feedback controller. Also presented is a novel control design that regulates the pass band of the considered micro beam. El-Ganaini. [9] Found an analytical and numerical study to investigate the vibration and stability of the Van der Pol equation subjected to external and parametric excitation forces via feedback control. The stability of the system is investigated applying the Lyapunov first method. The stability of the system is investigated applying Lyapunov first method.

El-Ganaini et al. [10] proposed a time-delayed positive position feedback controller to reduce and control the horizontal vibration of a magnetically levitated body subjected to multi force excitations. They exercised the method of multiple scales perturbation technique to obtain an approximate solution that clarifies the nonlinear behavior for both amplitude and phase of the whole system and shows the effects of time delay on the system. Zhao and Xu, [11] used delayed feedback control to restrain or stabilize the vibration of the primary system in a two-degree-of-freedom dynamical system with a parametrically excited the pendulum, studied internal resonance between pendulum and primary system and researched the effect of gain and delay on the vibration repression. As the delay converts at a fixed value of the gain, the vibration of the primary system can be suppressed at some values of the delay. The gain and delay could be chosen as the controlling parameters. Numerical simulation is convention, with the analytical solutions well.

Jun et al. [12] applied a nonlinear saturation controller NSC with a van der pol oscillator and additionally investigated the influence of feedback gains by using perturbation and direct numerical integration solutions. Gao and Chen [13] studied nonlinear analysis, design and vibration isolation for a bilinear system with time-delayed cubic velocity feedback. Gao and Chen [14] studied extensively the vibration control of many systems with the time delay by using different controllers. An active linear absorber based on positive position feedback control strategy has been developed and applied to suppress the high-amplitude response of a flexible beam subjected to a primary external excitation. El-Ganaini et al. [15] these researchers proposed a feedback controller of the positive time-delay position to reduce the horizontal vibration of the magnetic body subject to multi-force stimulation. This console is associated with the main system with 1: 1 internal ring.

Han et al. [16] interested with the designing and vibration control problem for networked nonlinear vehicle active suspension (NNVAS) with actuator time delay. Inserted in vehicle communication network to active commentary, a novel model for NNVAS has been established based on the Takagi-Sugeno fuzzy fusion technology. They have designed a reduced-order observer to solve the physically unrealizable problem of road disturbances. Rath et al. [17] have proposed a feedback active control suspension scheme to achieve ride comfort while maintaining the vehicle's holding path on the road .First, they have estimated the states of the nonlinear system by using a speed high observer where the suspension stroke is the only measurable output. Then they designed the controller by using a recursive derivative nonsingular higher order terminal sliding mode approach that avoids singularity.

Kocak and Ergenc [18] introduced a new approach to structure a delayed resonator structure with acceleration feedback, where they have modified a classical delayed resonator to an observer-based structure. Silva-Navarro and Abundis-Fong. [19] have studied and experimental evaluation of a passive/active cantilever beam autoparametric vibration absorber a two-story building-like structure (primary system), with two rigid floors connected by flexible columns. Sayed and Kamel [20] investigated an active vibration absorber for suppressing the vibration of the non-linear plant when subjected to external and parametric excitations in the presence of 1:2 and 1:3 internal resonance. They used the saturation phenomenon and internal resonance to control the steady state and transient vibrations. The numerical result shows that the saturation control of steady state vibrations is efficient. The stability of the obtained numerical solution is studied using both phase plane methods and frequency response equations.

El-Gohary and El-Ganaini [21] studied the vibration suppression of a damped structure subject to multi-parametric excitation forces. The model is represented by a two-degree-of-freedom system consisting of the main system and the absorber. They applied the multiple time scale perturbation to get an approximate solution up to the second order. The stability of the system is investigated numerically applying both phase-plane and frequency response functions. They studied the effects of different parameters of the absorber on system behavior numerically. Leung et. al. [22] analyzed the steady state bifurcation of a periodically excited system, in which three kinds of delayed feedback controls are considered to discuss the effects.

Zhao and Xu, [23] applied the feedback control and saturation control to suppress the vibration of the primary system in a two-degree-of-freedom dynamical system with a parametrically excited pendulum and showed that the delayed feedback control could be used to suppress the vibration or stabilize the system when the saturation control was invalid. The vibration of the primary system can be suppressed by the delayed feedback control when the original system was in the single-mode motion.

Kamel et al. [24] investigated the coupling of two non-linear oscillators of the main system and an absorber representing the ultrasonic cutting process. They controlled the main system behavior at simultaneous primary and internal resonance condition, where the system damage is probable. Eissa et al. [25] considered active suppression of nonlinear vibrations applying saturation-based controllers. They studied its effect on the system's behavior. Time delay inherently exists in many active control systems as a result of transport delay, online computation, measurements of the system states, executing the control algorithms, and processing of the errors and control signals.

In our work, we find that adding the absorber (time-delayed velocity feedback) minimizes the amplitude of vibration in the steady state, so we can control the effective stiffness associated with the passive absorber. The problem has been solved by using the Runge-kutta method. Effects of different parameters on the system behavior are studied numerically by using the MATLAB program. Finally, a comparison of previously published work is done at the end of this work.

II. MATHEMATICAL MODELING

Fig.1 the mechanical system schematic diagram. The system of primary concludes the system of linear spring mass with viscous damping and it is excited by an external harmonic force F (t) $= F_0 \cos\Omega t$, with amplitude and excitation frequency Ω. With respect to relieve the harmonic vibrations produced by F (t) vibration absorber of an auto parametric cantilever-beam is used. The nonlinear absorber is collected by a thin beam joined the primary system and with an equivalent mass m at the end with lateral motion confined to the plane which is horizontal. The length l indicates the total length of the beam and c2 is a small viscous damping on the beam. There are two subsystems primary and secondary which are coupled by the inertia resulted means from the beam which is attachment, besides, due to the entire system requires any kind of actuators, it results in a purely passive vibration control scheme.

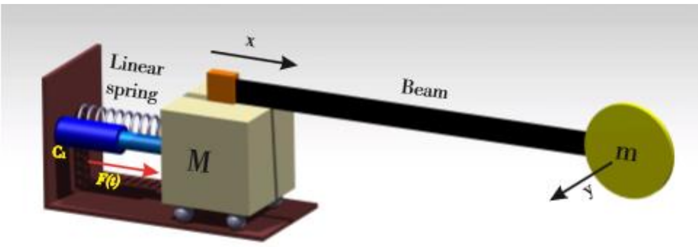


Figure 1: schematic diagram of the system without absorber

III. EQUATION OF MOTION WITH PPF CONTROL

The non-linear dynamical system is consists of external force .The system is represented by a two-degree-of-freedom (2dof) coupled and differential equations represented by the main system and absorber. From the principles of the mechanics the derived equation of motion can be written the forms Eqs (1) and (2) [26].

$$\ddot{x}+2\varsigma_1\omega_1\dot{x}+\omega_1^2 x-h(y\ddot{y}+\dot{y}^2)=f\cos(\Omega t)-R_1\dot{x}(t-\tau) \quad (1)$$

$$\ddot{y}+2\varsigma_2\omega_2\dot{y}+(\omega_2^2-g\ddot{x})y+\beta y(y\ddot{y}+\dot{y}^2)=-R_2\dot{y}(t-\tau) \quad (2)$$

where x and y denote the longitudinal motion of primary system and lateral displacement of the passive cantilever beam absorber , respectively, ε small perturbation parameter, ω_1 is natural parametric frequency of the primary system, ω_2 is natural frequency nonlinear absorber, ς_1,ς_2 are damping factor ,Ω is excitation frequency ,f external force, τ actuation delay.

IV. MATHEMATICAL ANALIYSIS

Eqs (1) and (2) can be solved analytically using multiple time scale perturbation technique as:

$$x(t,\varepsilon)=x_0(T_0,T_1)+\varepsilon x_1(T_0,T_1)+\varepsilon^2 x_2(T_0,T_1)+\ldots \quad (3)$$

$$y(t,\varepsilon)=y_0(T_0,T_1)+\varepsilon y_1(T_0,T_1)+\varepsilon^2 y_2(T_0,T_1)+\ldots \quad (4)$$

$$x(t-\tau,\varepsilon)=x_{0\tau}(T_0,T_1)+\varepsilon x_{1\tau}(T_0,T_1)+\varepsilon^2 x_{2\tau}(T_0,T_1)+\ldots \quad (5)$$

$$y(t-\tau,\varepsilon)=y_{0\tau}(T_0,T_1)+\varepsilon y_{1\tau}(T_0,T_1)+\varepsilon^2 y_{2\tau}(T_0,T_1)+\ldots \quad (6)$$

Where T_0 =t is fast time scale, which is associated with changes occurring at the frequencies, Ω and T_1= εt is the slow time scale, which is associated with modulations in the amplitudes and phases resulting from the non-linearity's and parametric resonance. In term of T_0 and T_1 the time derivatives became

$$\frac{d}{dt}=D_0+\varepsilon D_1+\varepsilon^2 D_2+\ldots \quad (7)$$

$$\frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0 D_1+\varepsilon^2(D_1^2+2D_0 D_2)+\ldots \quad (8)$$

Where D_n differential operators; $D_n=\frac{\partial}{\partial T_n}$ (n=0, 1).To obtains a uniformly valid approximate solution of this system we order the dimensionless parameters of system by the formal small parameter ε as following:

$$\varsigma_1=\varepsilon\hat{\varsigma}_1,\ \varsigma_2=\varepsilon\hat{\varsigma}_2,\ h=\varepsilon\hat{h},\ R_1=\varepsilon\hat{R}_1,\ R_2=\varepsilon\hat{R}_2,\ g=\varepsilon\hat{g},\ \beta=\varepsilon^2\hat{\beta},\ f=\varepsilon\hat{f} \quad (9)$$

Substituting Eqs (3 -6) into Eqs (1) and (2), and equate the coefficients of power of ε we obtain the following:

Order (ε^0):

$$(D_0^2+\omega_1^2)x_0=0 \quad (10)$$

$$(D_0^2+\omega_2^2)y_0=0 \quad (11)$$

Order (ε^1):

$$(D_0^2+\omega_1^2)x_1=-2D_0D_1x_0-2\hat{\varsigma}_1\omega_1 D_0 x_0+\hat{h}y_0D_0^2y_0+\hat{h}(D_0y_0)^2+\hat{f}\cos(\Omega t)-\hat{R}_1D_0x_{0\tau} \quad (12)$$

$$(D_0^2+\omega_2^2)y_1=-2D_0D_1y_0-2\hat{\varsigma}_2\omega_2D_0y_0+\hat{g}y_0D_0^2x_0-\hat{R}_2D_0y_{0\tau} \quad (13)$$

The general solution of Eqs (9) and (10) can be expressed in the form:

$$x_0(T_0,T_1)=A_0 e^{i\omega_1 T_0}+cc \quad (14)$$

$$y_0(T_0,T_1)=B_0 e^{i\omega_2 T_0}+cc \quad (15)$$

where A_0 and B_0 are unknown function in T_1, which can be determined by imposing the solvability condition at the next approximation order by eliminating the secular and small- divisor terms.

$$x_{0\tau}(T_0,T_1)=A_{0\tau}(T_1)e^{i\omega_1(T_0-\tau)}+cc \quad (16)$$

$$y_{0\tau}(T_0,T_1) = B_{0\tau}(T_1)e^{i\omega_2(T_0-\tau)} + cc \tag{17}$$

Expanding $A_{0\tau}$ and $B_{0\tau}$ in Taylor series, we get:

$$A_{0\tau}(T_1) = A_0(T_1-\varepsilon\tau) \cong A_0(T_1) - \varepsilon\tau A_0'(T_1) + O(\varepsilon^2) \tag{18}$$

$$B_{0\tau}(T_1) = B_0(T_1-\varepsilon\tau) \cong B_0(T_1) - \varepsilon\tau B_0'(T_1) + O(\varepsilon^2) \tag{19}$$

Where A_0', B_0' prime donates derivative T_1 .Substituting Eqs (14-17) into Eqs (12) and (13) we get:

$$\left(D_0+\omega_1^2\right)x_1 = -2i\omega_1 D_1\left(A_0e^{i\omega_1T_0}-\bar{A}_0e^{-i\omega_1T_0}\right)-2i\omega_1^2\hat{\varsigma}_1\left(A_0e^{i\omega_1T_0}-\bar{A}_0e^{-i\omega_1T_0}\right)-i\omega_1\hat{R}_1A_0e^{i\omega_1(T_0-\tau)}$$
$$+i\varepsilon\omega_1\tau\hat{R}_1D_1A_0e^{i\omega_1(T_0-\tau)T_0}-2\omega_2^2\hat{h}\bar{B}_0^2e^{-2i\omega_2T_0}-2\omega_2^2\hat{h}B_0^2e^{2i\omega_2T_0}+\frac{\hat{f}}{2}\left(e^{i\Omega T_0}+e^{-i\Omega T_0}\right) \tag{20}$$

$$\left(D_0+\omega_2^2\right)y_1 = -2i\omega_2 D_1\left(B_0e^{i\omega_2T_0}-B_0e^{-i\omega_2T_0}\right)-2i\omega_2^2\hat{\varsigma}_2\left(B_0e^{i\omega_2T_0}-\bar{B}_0e^{-i\omega_2T_0}\right)-i\omega_2\hat{R}_2B_0e^{i\omega_2(T_0-\tau)}$$
$$+i\varepsilon\omega_2\tau\hat{R}_2D_1B_0e^{i\omega_2(T_0-\tau)}-\omega_1^2\hat{g}\left(A_0B_0e^{i(\omega_1+\omega_2)T_0}+\bar{A}_0\bar{B}_0e^{-i(\omega_1+\omega_2)T_0}\right)$$
$$-\omega_1^2\hat{g}\left(A_0\bar{B}_0e^{i(\omega_1-\omega_2)T_0}+B_0\bar{A}_0e^{-i(\omega_1-\omega_2)T_0}\right) \tag{21}$$

After eliminating the secular terms, the general solution of equation (20) and (21) is given by:

$$x_1(T_0,T_1) = A_1e^{i\omega_1T_0}+\frac{\hat{f}}{2(\omega_1^2-\Omega^2)}e^{i\Omega T_0}-\frac{2\omega_2^2\hat{h}B_0^2}{(\omega_1^2-4\omega_2^2)}e^{2i\omega_2T_0}+cc \tag{22}$$

$$y_1(T_0,T_1) = B_1e^{i\omega_2T_0}-\frac{\hat{g}\omega_1^2A_0B_0}{(\omega_2^2-(\omega_1+\omega_2)^2)}e^{i(\omega_1+\omega_2)T_0}-\frac{\omega_1^2\hat{g}A_0\bar{B}_0}{(\omega_2^2-(\omega_1-\omega_2)^2)}e^{i(\omega_1-\omega_2)T_0}+cc \tag{23}$$

Where (Γ_i , i=1…4) and A_1 , B_1 are complex function in T_1, and cc is complex conjugate of the preceding terms.

V. Stability analysis

After numerically studying the different resonance cases and deduce the worst ones, one of the worst cases has been chosen to study the system stability. The selected resonance case are combine between the Primary resonance and internal resonance ($\omega_1=\Omega, \omega_1=2\omega_2$). In this case we introduce the detuning parameter σ_1, σ_2 according to:

$$\Omega = \omega_1+\sigma_1 = \omega_1+\varepsilon\hat{\sigma}_1 \tag{24}$$
$$\omega_1 = 2\omega_2+\sigma_2 = 2\omega_2+\varepsilon\hat{\sigma}_2 \tag{25}$$

Where σ_1, σ_2 are the detuning parameters .Also for stability investigation, the analysis is limited to the first approximation . So, our solution is only dependent on T_0 and T_1. Substituting Eqs (24) and (25) into Eqs. (20) and (21) and eliminating the secular terms leads to the solvability conditions

$$\left(-2i\omega_1 D_1 A_0 - 2i\omega_1^2 \hat{\varsigma}_1 A_0 - i\omega_1 A_0 \hat{R}_1 e^{-i\omega_1\tau} + i\varepsilon\omega_1 \tau R_1 D_1 A_0 e^{-i\omega_1\tau} - 2\omega_2^2 \hat{h} B_0^2 e^{-i\varepsilon\hat{\sigma}_2 T_1} + \frac{f}{2} e^{i\varepsilon\hat{\sigma}_1 T_1}\right) = 0 \tag{26}$$

$$\left(-2i\omega_2 D_1 B_0 - 2i\omega_2^2 \hat{\varsigma}_2 B_0 - i\omega_2 B_0 \hat{R}_2 e^{-i\omega_2\tau} + i\varepsilon\omega_2 \tau \hat{R}_2 D_1 B_0 e^{-i\omega_2\tau} - \omega_1^2 \hat{g}\hat{h} A_0 \overline{B}_0 e^{i\varepsilon\hat{\sigma}_2 T_1}\right) = 0 \tag{27}$$

To analyze the solution of Eqs (26) and (27), it is convenient to express A in the polar form as:

$$A_0 = \frac{1}{2} a_1(T_1) e^{i\theta_1(T_1)}, D_1 A_0 = \frac{1}{2} a_1' e^{i\theta_1} + \frac{i}{2}\theta_1' a_1 e^{i\theta_1} \tag{28}$$

$$B_0 = \frac{1}{2} a_2(T_1) e^{i\theta_2(T_1)}, D_1 B_0 = \frac{1}{2} a_2' e^{i\theta_2} + \frac{i}{2}\theta_2' a_2 e^{i\theta_2} \tag{29}$$

Where $a_1, a_2, \theta_i (i=1,2)$ are unknown real-valued function .Inserting Eqs (28) and (29) into Eqs (26) and (27) and separating the real and imaginary parts we have the following:

$$E_1 \dot{a}_1 = -\omega_1 \varsigma_1 a_1 + \frac{\omega_2^2 h a_2^2}{2\omega_1} \sin\gamma_2 - \frac{a_1 R_1}{2}\cos(\omega_1\tau) + \frac{\tau R_1}{2} a_1 \dot{\theta}_1 \sin(\omega_1\tau) + \frac{f}{2\omega_1}\sin\gamma_1 \tag{30}$$

$$E_1 a_1 \dot{\theta}_1 = \frac{\omega_2^2 h a_2^2}{2\omega_1}\cos\gamma_2 + \frac{a_1 R_1}{2}\sin(\omega_1\tau) - \frac{\tau R_1}{2}\dot{a}_1 \sin(\omega_1\tau) - \frac{f}{2\omega_1}\cos\gamma_1 \tag{31}$$

$$E_2 \dot{a}_2 = -\omega_2 \varsigma_2 a_2 - \frac{\omega_1^2 g a_1 a_2}{4\omega_2}\sin\gamma_2 - \frac{R_2}{2} a_2 \cos(\omega_2\tau) + \frac{\tau R_2}{2} a_2 \dot{\theta}_2 \sin(\omega_2\tau) \tag{32}$$

$$E_2 a_2^2 \dot{\theta}_2 = \frac{\omega_1^2 g a_1 a_2}{4\omega_2}\cos\gamma_2 + \frac{R_2}{2} a_2 \sin(\omega_2\tau) - \frac{\tau R_2}{2}\dot{a}_2 \sin(\omega_2\tau) \tag{33}$$

Where dot represent derivative

$$\left.\begin{aligned} \gamma_1 &= \varepsilon\hat{\sigma}_1 T_1 - \theta_1 = \sigma_1 T_1 - \theta_1 \\ \gamma_2 &= \varepsilon\hat{\sigma}_2 T_1 + \theta_1 - 2\theta_2 = \sigma_2 T_1 + \theta_1 - 2\theta_2 \end{aligned}\right\} \tag{34}$$

Differentiate (34) to obtain

$$\left.\begin{aligned} \dot{\theta}_1 &= \sigma_1 - \dot{\gamma}_1 \\ \dot{\theta}_2 &= \frac{1}{2}(\sigma_2 + \sigma_1) - \frac{1}{2}(\dot{\gamma}_1 + \dot{\gamma}_2) \end{aligned}\right\} \tag{35}$$

For steady solutions $\dot{a}_1 = \dot{a}_2 = 0, \dot{\gamma}_i = 0$ and the periodic solution at the fixed points corresponding and insert (35) to Eqs (30)-(33) is given by:

$$\frac{f}{2\omega_1}\sin\gamma_1 = \omega_1\varsigma_1 a - \frac{\omega_2^2 h a_2^2}{2\omega_1}\sin\gamma_2 + \frac{1}{2} a_1 R_1 \cos(\omega_1\tau) - \frac{\tau}{2} a_1 R_1 \sigma_1 \sin(\omega_1\tau) \tag{36}$$

$$\frac{f}{2\omega_1}\cos\gamma_1 = -\sigma_1 a + \frac{\omega_2^2 h b^2}{2\omega_1}\cos\gamma_2 + \frac{1}{2} a_1 R_1 \sin(\omega_1\tau) + \frac{\tau}{2}\sigma_1 a_1 R_1 \cos(\omega_1\tau) \tag{37}$$

$$\frac{\omega_1^2 g a_1 a_2}{4\omega_2}\sin\gamma_2 = -\omega_2\varsigma_2 a_2 - \frac{1}{2} a_2 R_2 \cos(\omega_2\tau) + \frac{\tau R_2}{4} a_2 (\sigma_1 + \sigma_2)\sin(\omega_2\tau) \tag{38}$$

$$\frac{1}{2} a_2 (\sigma_1 + \sigma_2) = \frac{\omega_1^2 g a_1 a_2}{4\omega_2}\cos\gamma_2 + \frac{1}{2} a_2 R_2 \sin(\omega_2\tau) + \frac{\tau R_2}{4} a_2 (\sigma_1 + \sigma_2)\cos(\omega_2\tau) \tag{39}$$

From Eqs (36)-(39) we get the corresponding frequency response equation (FRE) is:

$$a_1^2=\left(-\frac{4\omega_2^2\varsigma_2}{\omega_1^2 g}-\frac{2\omega_2 R_2}{\omega_1^2 g}\cos(\omega_2\tau)+\frac{\omega_2\tau R_2}{\omega_1^2 g}(\sigma_1+\sigma_2)\sin(\omega_2\tau)\right)^2+\left(\frac{2\omega_2(\sigma_1+\sigma_2)}{\omega_1^2 g}-\frac{2\omega_2 R_1}{\omega_1^2 g}\sin(\omega_2\tau)\right.$$

$$\left.-\frac{\omega_2\tau R_2}{\omega_1^2 g}(\sigma_1+\sigma_2)\cos(\omega_2\tau)\right)^2 \tag{40}$$

$$\left(\frac{f}{2\omega_1}\right)^2=\left(\omega_1\varsigma_1 a_1+\frac{\omega_2^4 h\varsigma_2 a_2^2}{\omega_1^3 g a_1}+\frac{\omega_2^3 hR_2 a_2^2}{\omega_1^3 g a_1}\cos(\omega_2\tau)+\frac{1}{2}a_1 R_1\cos(\omega_1\tau)-\frac{\tau R_1}{2}a_1\sigma_1\sin(\omega_1\tau)\right.$$
$$\left.-\frac{\omega_2^3\tau hR_2 a_2^2}{2\omega_1^3 g a_1}(\sigma_1+\sigma_2)\sin(\omega_2\tau)\right)^2+\left(-\sigma_1 a_1+\frac{\omega_2^3 h a_2^2(\sigma_1+\sigma_2)}{\omega_1^3 g a_1}-\frac{\omega_2^3 hR_2 a_2^2}{\omega_1^3 g a_1}\sin(\omega_2\tau)\right.$$
$$\left.+\frac{1}{2}a_1 R_1\sin(\omega_1\tau)-\frac{\omega_2^3\tau hR_2 a_2^2}{2\omega_1^3 g a_1}(\sigma_1+\sigma_2)\cos(\omega_2\tau)+\frac{\tau R_1}{2}a_1\sigma_1\cos(\omega_1\tau)\right)^2 \tag{41}$$

VI. Linear solution

To study the stability of the linear solution of the obtained fixed points, let us consider A and B in the forms

$$A_0=\frac{1}{2}(p_1-iq_1)e^{i\delta_1 T_1} \tag{42}$$
$$B_0=\frac{1}{2}(p_2-iq_2)e^{i\delta_2 T_1} \tag{43}$$

where p_1, q_1, p_2 *and* q_2 are real values and considering $\delta_1=\sigma_1$.
Substituting from Eqs (40) and (41) into the linear parts of Eqs (26) and (27) and separating real and imaginary parts, the following system of equations is obtained:

$$p_1'=\left(\frac{M_8}{M_7}\right)p_1+\left(\frac{M_9}{M_7}\right)q_1 \tag{44}$$

$$q_1'=\left(-\frac{M_2}{E_1}+\frac{E_4 M_8}{E_1 M_7}\right)p_1+\left(\frac{M_1}{E_1}+\frac{E_4 M_9}{E_1 M_7}\right)q_1 \tag{45}$$

$$p_2'=\left(\frac{M_{11}}{M_{10}}\right)p_2+\left(\frac{M_{12}}{M_{10}}\right)q_2 \tag{46}$$
$$q_2'=\left(-\frac{M_5}{E_2}+\frac{E_6 M_{11}}{E_2 M_{10}}\right)p_2+\left(\frac{M_4}{E_2}+\frac{E_6 M_{12}}{E_2 M_{10}}\right)q_2 \tag{47}$$

The above equations can be written in a matrix form as:

$$\begin{bmatrix} p_1' \\ q_1' \\ p_2' \\ q_2' \end{bmatrix} = \begin{bmatrix} \left(\frac{M_8}{M_7}\right) & \left(\frac{M_9}{M_7}\right) & 0 & 0 \\ \left(-\frac{M_2}{E_1}+\frac{E_4M_8}{E_1M_7}\right) & \left(\frac{M_1}{E_1}+\frac{E_4M_9}{E_1M_7}\right) & 0 & 0 \\ 0 & 0 & \left(\frac{M_{11}}{M_{10}}\right) & \left(\frac{M_{12}}{M_{10}}\right) \\ 0 & 0 & \left(-\frac{M_5}{E_2}+\frac{E_6M_{11}}{E_2M_{10}}\right) & \left(\frac{M_4}{E_2}+\frac{E_6M_{12}}{E_2M_{10}}\right) \end{bmatrix} \begin{bmatrix} p_1 \\ q_1 \\ p_2 \\ q_2 \end{bmatrix} \tag{48}$$

The stability of the linear solution in this case is obtained from the zero characteristic equation

$$\begin{vmatrix} -\left(\lambda+\left(\frac{M_8}{M_7}\right)\right) & \left(\frac{M_9}{M_7}\right) & 0 & 0 \\ \left(-\frac{M_2}{E_1}+\frac{E_4M_8}{E_1M_7}\right) & -\left(\lambda+\left(\frac{M_1}{E_1}+\frac{E_4M_9}{E_1M_7}\right)\right) & 0 & 0 \\ 0 & 0 & -\left(\lambda+\left(\frac{M_{11}}{M_{10}}\right)\right) & \left(\frac{M_{12}}{M_{10}}\right) \\ 0 & 0 & \left(-\frac{M_5}{E_2}+\frac{E_6M_{11}}{E_2M_{10}}\right) & -\left(\lambda+\left(\frac{M_4}{E_2}+\frac{E_6M_{12}}{E_2M_{10}}\right)\right) \end{vmatrix} = 0 \tag{49}$$

After extract we obtain that:

$$\lambda^4 + r_1\lambda^3 + r_2\lambda^2 + r_3\lambda + r_4 = 0 \tag{50}$$

Where r_1, r_2, r_3 and r_4 are defined in Appendix.

According to Routh-Huriwitz criterion, the above linear solution is stable if the following inequalities are satisfied:

$$r_1 > 0, r_1 r_2 - r_3 > 0, r_3(r_1 r_2 - r_3) - r_1^2 r_4 > 0, r_4 > 0$$

VII. Non-linear solution

To determine the stability of the fixed points, one lets

$$a_1 = a_{10} + a_{11}, a_2 = a_{20} + a_{21}, \gamma_m = \gamma_{m0} + \gamma_{m1} \left(m = 1, 2\right) \tag{51}$$

Where a_{10}, a_{20} and γ_{m0} are solutions of Eqs (36) - (39) and $a_{11}, a_{21}, \gamma_{m1}$ are perturbations which are assumed to be small compared to a_{10}, a_{20} and γ_{m0}. Substituting Eq (51) into Eqs (30)-(33) using Eqs (36) - (39) and keeping only the linear terms, we obtain:

$$\dot{a}_1 = \Gamma_{11}a_{11} + \Gamma_{12}\gamma_{11} + \Gamma_{13}a_{21} + \Gamma_{14}\gamma_{21} \tag{52}$$

$$\dot{\gamma}_{11} = \Gamma_{21}a_1 + \Gamma_{22}\gamma_{11} + \Gamma_{23}a_{21} + \Gamma_{24}\gamma_{21} \tag{53}$$

$$\dot{a}_2 = \Gamma_{31}a_{11} + \Gamma_{32}a_{21} + \Gamma_{33}\gamma_{21} \tag{54}$$

$$\dot{\gamma}_{21} = \Gamma_{41}a_{11} + \Gamma_{42}\gamma_{11} + \Gamma_{43}a_{21} + \Gamma_{43}\gamma_{21} \tag{55}$$

Where $\Gamma_{ij}, (i=1...4), (j=1...4)$ are functions defined in Appendix.

VIII. Numerical results and discussion

To study behavior of the main system numerically the (Rung-Kutta method) of the nonlinear system, given by Eqs (1) and (2) at basic without absorber, the primary and internal resonance cases($\Omega = \omega_1, \omega_1 = 2\omega_2$) are obtained as shown in figures (2)-(4).these solutions are obtained at selected values (Ω =2.5, $\Omega = \omega_1 = 2\omega_2$).

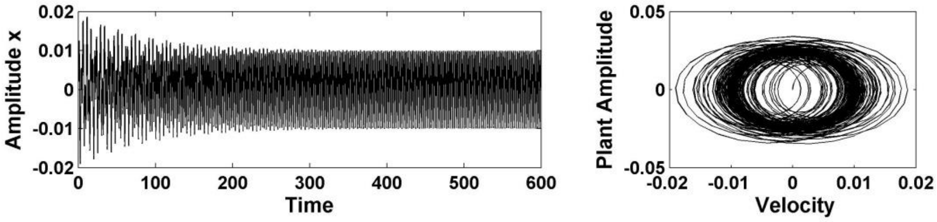


Figure2: Response of the system without absorber at basic case

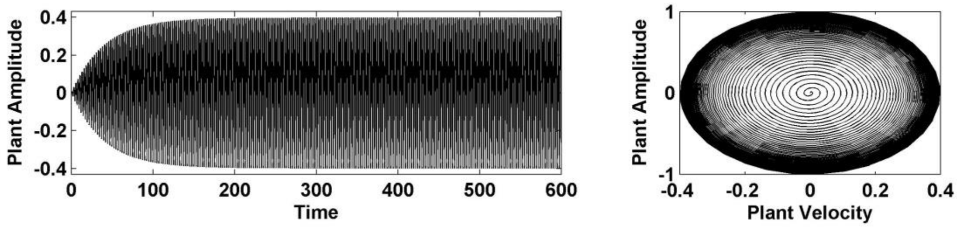


Figure3: Response of the system at resonance case ($\Omega = \omega_1, \omega_1 = 2\omega_2$)

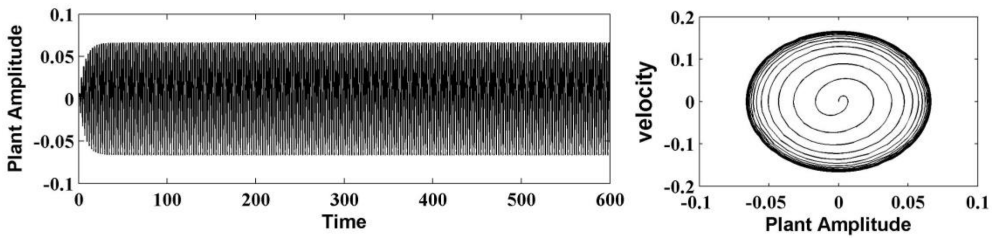


Figure4: Response of the system with absorber in resonance case ($\Omega = \omega_1, \omega_1 = 2\omega_2$)

Fig. (2) Show that study of amplitude on the main system without absorber of the selection of values as ($f = 0.05$, $\varsigma_1 = 0.01$, $\omega_1 = 1.1$, $\Omega = 2.5$).Fig.(3) Study the amplitude in the system with absorber at resonance case ($\Omega = \omega_1$)we find that in this case the amplitude at maximum reached (up to approximately 0.38),Fig.(4) Show the effect of time delay at the response case, we find absorber able to reduce and control vibration significantly until amplitude reached (up to approximately 0.055).

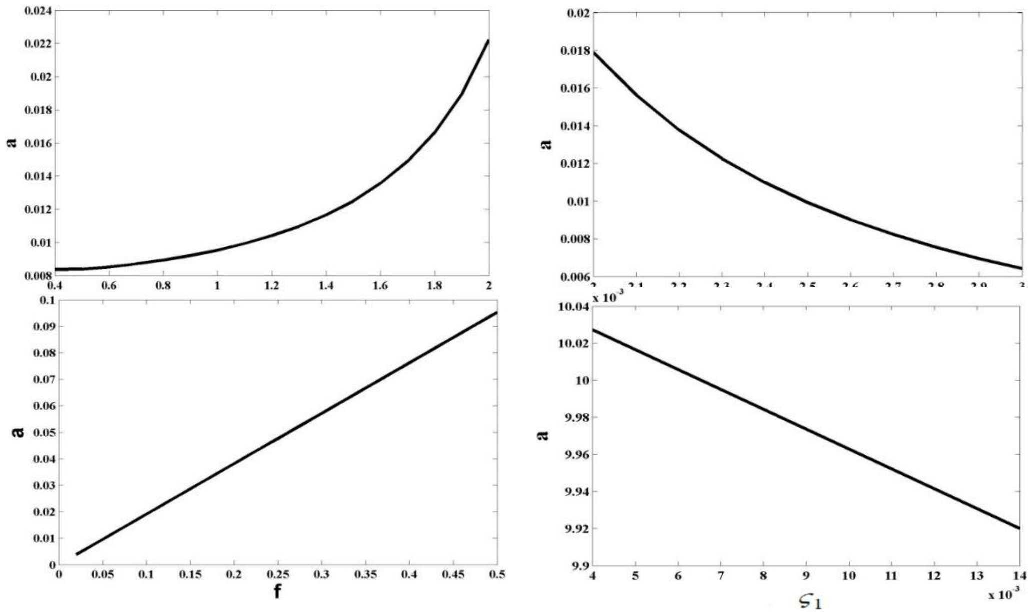
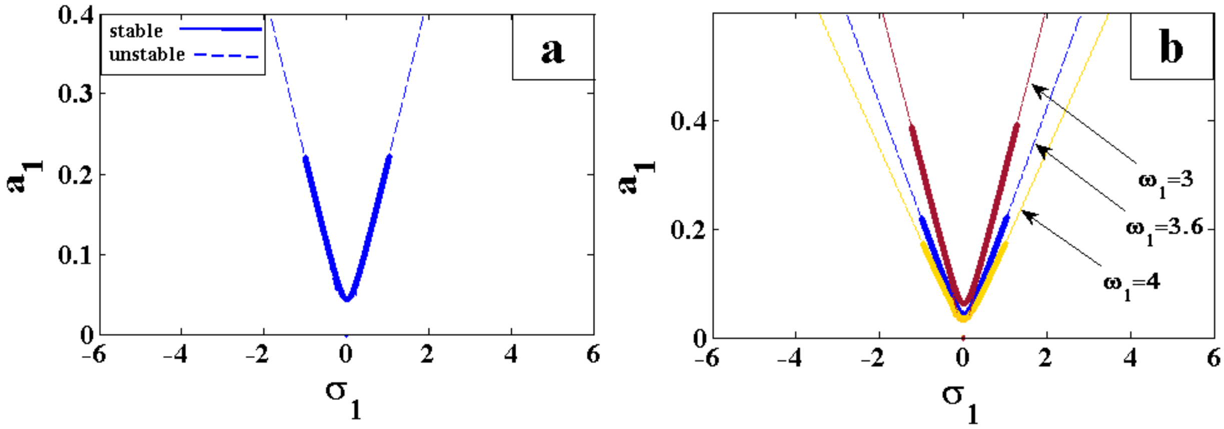


Figure5: Effect of parameters on amplitude of the system without absorber

fig.5 show effect of different parameter on the main system without absorber .can see amplitude increasing as f, ω_1 are increased **.**Also when decreasing the values Ω , ς_1 are increased as shown

Theoretical frequency and force response curve

The frequency equation is represented graphically by using the numerical methods. The frequency response equation is nonlinear algebraic equation, which are solved numerically by using Newton Raphson method .frequency response equation (19) and (20) is nonlinear algebraic equation, the results are shown in figure (6) for the steady state amplitudes a_1 against parameter σ_1 and figure (7) for the steady state amplitudes a_2 against parameter σ_1



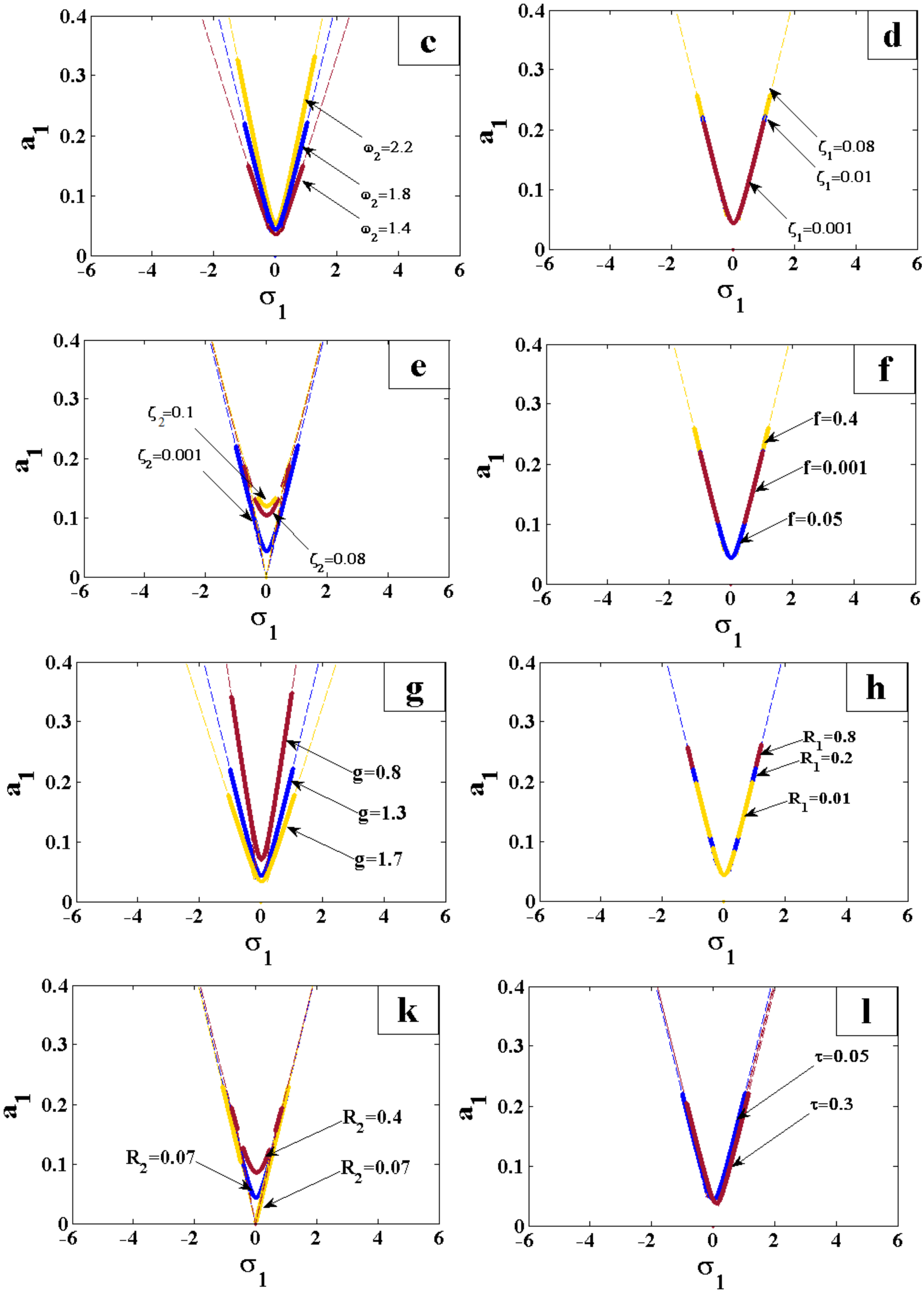
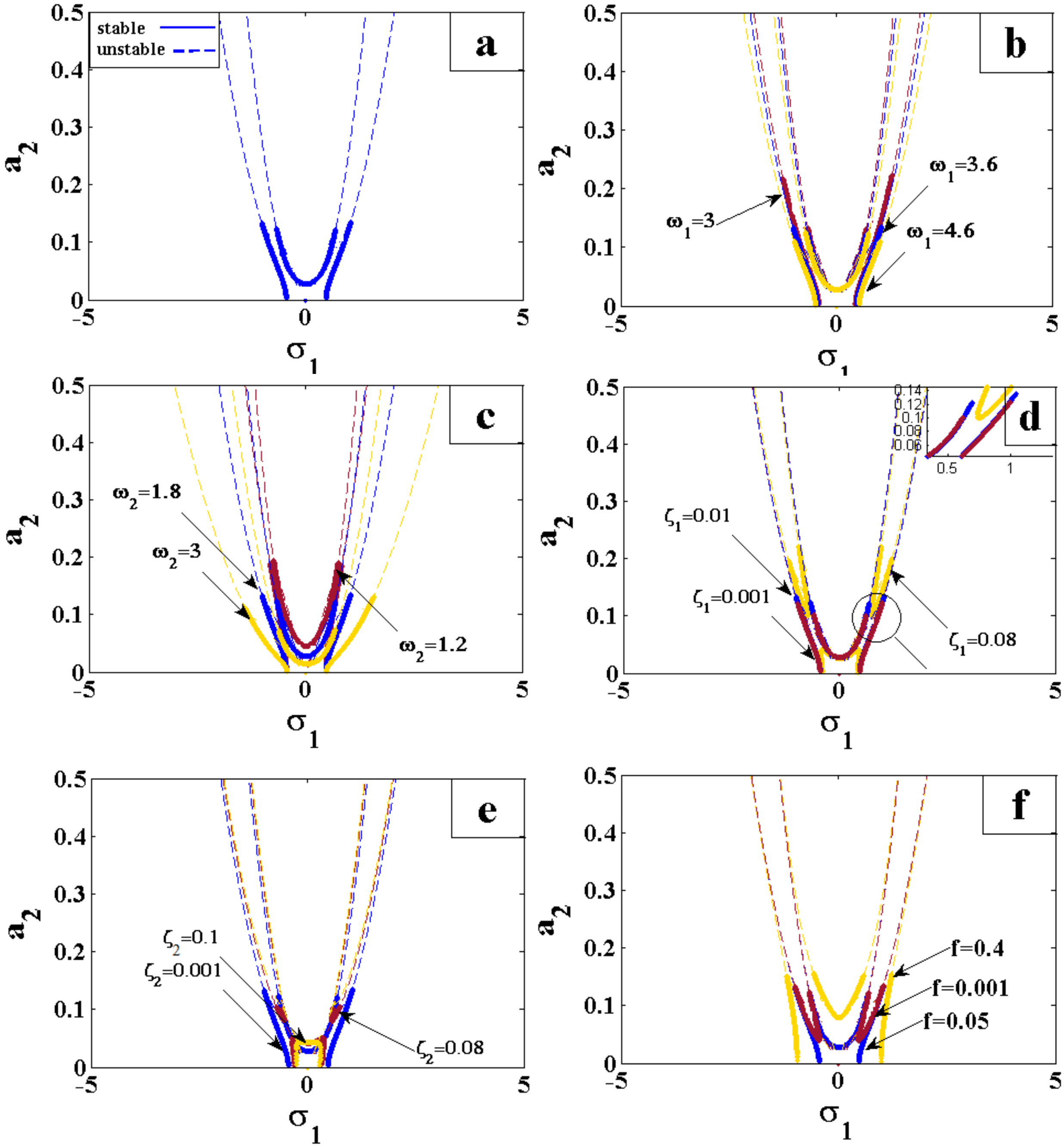


Figure7: Stability of the practical case, σ_2 =0 on steady state amplitude a_1 against σ_1

Fig.7 show effect of parameter on the system at the steady-state amplitude a_1 increased when $(\omega_2, \varsigma_1, \varsigma_2, f)$ increasing and amplitude value decreased when $(\omega_1, g, R_1, R_2, \tau)$ increasing. In the curve (a) shows the relation a_1 and σ_1 it shows that the sable in the bottom branch and unstable in the up branch, the period of stability and un-stability changes with the study of parameters as is shown



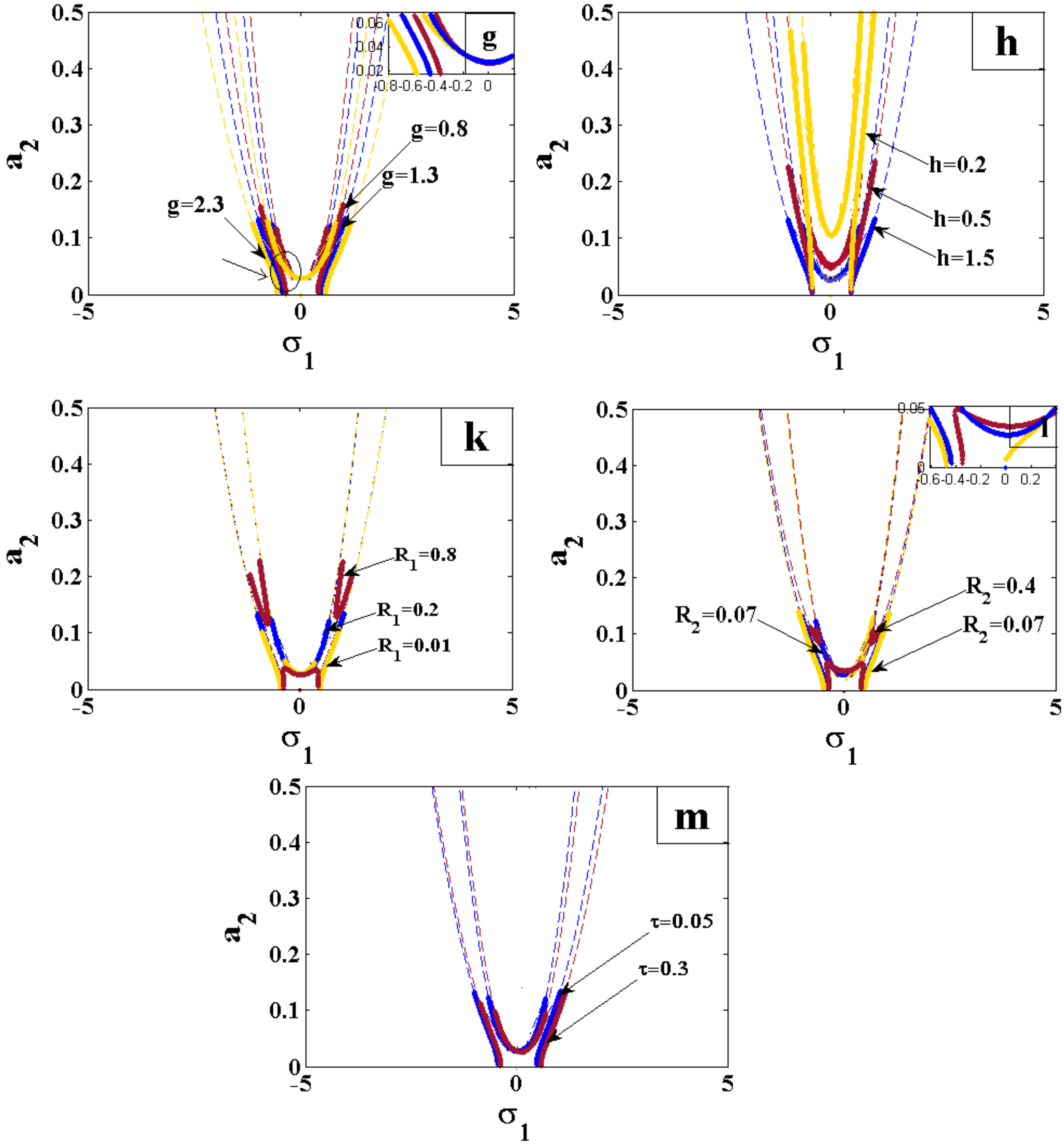


Figure8: Stability of the practical case, σ_2 =0 on steady state amplitude a_2 against σ_1

Fig.8 show effect of parameter on the system at the steady-state amplitude a_1 increased when $(\varsigma_1, \varsigma_2, f, R_1, R_2)$ increasing and decreasing ω_2 and amplitude value decreased when (ω_1, g, h, τ) increasing. In the curve (a) shows the relation a_2 and σ_1 it shows that the sable in the bottom branch and unstable in the up branch, the period of stability and un-stability changes with the study of parameters as is shown

IX. Conclusion

we studying the vibration numerically on the system with and without control .To study the stability of the system obtained numerical solution is investigated using both phase plane methods and frequency response equation in conjunction with study of resonance case an addition to studying effect of time delay on the system to reduce the amplitude and control on vibration, both frequency response equation and results based on the present investigation the above study the following conclusions are shown:

1- The study of resonance cases numerically we conclude that the worst cases simultaneous resonance $(\Omega = \omega_1),(\omega_1 = 2\omega_2)$ on the system with absorber and without and comparison of the absorber effect on both cases.

2- The behavior of amplitude on the worst case with control is more stable and control vibration is almost nonexistent than system without control, where the effect of absorber reduce the amplitude from (0.38 nearly) to (0.055 nearly) in case $(\Omega = \omega_1),(\omega_1 = 2\omega_2)$.

3- The steady state amplitude of the main system is monotonic increasing function when increasing values $(\omega_2, \varsigma_1, \varsigma_2, f)$ be on steady state amplitude a_1 against σ_1, and monotonic decreasing function when increasing values $(\omega_1, g, R_1, R_2, \tau)$ on steady state amplitude a_1 against σ_1 .

4- The steady state amplitude of the main system is monotonic increasing function when increasing values $(\varsigma_1, \varsigma_2, f, R_1, R_2)$ and decreasing ω_2 be on steady state amplitude a_2 against σ_1, and monotonic decreasing function when increasing values ω_1, g, h, τ on steady state amplitude a_2 against σ_1 .

5- The relation a_1 and σ_1 it shows that the sable in the bottom branch and un-stable in the up branch, the period of stability and un-stability changes with the study of parameters and relation a_2 and it shows that the sable in the bottom branch on the curved and unstable in the up branch, the period of stability and instability changes with the study of parameters. And the effect of time delay on the system at the worst case very clear and by using this absorbent the vibration is controlled to maintain the use of the device safely without causing any damage or damage to the device.

Disclosure statement

The authors have disclosed that they do not have any conflicts of interest to declare.

Authors' contributions
Y.Amer : Conceptualization, Methodology, Agwa: Software, Writing- Original draft preparation **.**A.T. Elsayed: Writing- Reviewing and Editing, Data curation.

Availability of data and materials
The information applied in this research is ready from the authors at request.

Appendix

$$E_1=(1-\frac{\tau R_1}{2}\cos(\omega_1\tau))\ ,E_2=(1-\frac{\tau R_2}{2}\cos(\omega_2\tau)),E_3=(1+\frac{\tau^2 R_1^2}{4E_1^2}(\sin(\omega_1\tau))\wedge 2)$$

$$E_4=\frac{\tau R_1}{2}\sin(\omega_1\tau)\ ,E_5=(1+\frac{\tau^2 R_2^2}{8E_2^2}(\sin(\omega_2\tau))\wedge 2)\ ,E_6=\frac{\tau R_2}{2}\sin(\omega_2\tau)$$

$$M_1=(-\frac{1}{2}R_1\cos(\omega_1\tau)-\omega_1\varsigma_1+\frac{\tau R_1\delta_1}{2}\sin(\omega_1\tau))\ ,M_2=(\frac{1}{2}R_1\sin(\omega_1\tau)-\delta_1+\frac{\tau R_1\delta_1}{2}\cos(\omega_1\tau))$$

$$M_4=(-\omega_2\varsigma_2-\frac{1}{2}R_2\cos(\omega_2\tau)+\frac{\tau R_2\delta_2}{2}\sin(\omega_2\tau))\ ,M_5=(-\delta_2+\frac{1}{2}R_2\sin(\omega_2\tau)+\frac{\tau R_2\delta_2}{2}\cos(\omega_2\tau))$$

$$M_7=(E_1+\frac{E_4^2}{E_1})\ ,M_8=(M_1+\frac{E_4M_2}{E_1})\ ,M_9=(M_2-\frac{E_4M_1}{E_1})\ ,M_{10}=(E_2+\frac{E_6^2}{E_2})$$

$$M_{11}=(M_4+\frac{E_6M_5}{E_2})\ ,M_{12}=(M_5-\frac{E_6M_4}{E_2})$$

$$r_1=-(\frac{M_1}{E_1}+\frac{E_4M_9}{E_1M_7}+\frac{M_8}{M_7}+\frac{M_4}{E_2}+\frac{E_6M_{12}}{E_2M_{10}}+\frac{M_{11}}{M_{10}})$$

$$r_2=(\frac{M_{11}M_4}{E_2M_{10}}+\frac{M_5M_{12}}{E_2M_{10}}-\frac{M_2M_9}{E_1M_7}+\frac{E_4M_8M_9}{E_1M_7^2}+(\frac{M_1}{E_1}+\frac{E_4M_9}{E_1M_7}+\frac{M_8}{M_7})(\frac{M_4}{E_2}+\frac{E_6M_{12}}{E_2M_{10}}+\frac{M_{11}}{M_{10}}))$$

$$r_3=((\frac{M_1}{E_1}+\frac{E_4M_9}{E_1M_7}+\frac{M_8}{M_7})(-\frac{M_5M_{12}}{E_2M_{10}}-\frac{M_4M_{11}}{E_2M_{10}})-(\frac{M_4}{E_2}+\frac{E_6M_{12}}{E_2M_{10}}+\frac{M_{11}}{M_{10}})(\frac{M_1M_8-M_2M_9}{E_1M_7}+\frac{2E_4M_8M_9}{E_1M_7^2}))$$

$$r_4=((\frac{M_1M_8}{E_1M_7}+\frac{2E_4M_8M_9}{E_1M_7^2}-\frac{M_2M_9}{E_1M_7})(\frac{M_4M_{11}+M_5M_{12}}{E_2M_{10}}))$$

$$\Gamma_{11}=-\frac{\omega_1\varsigma_1}{E_1E_3}-\frac{R_1}{2E_1E_3}\cos(\omega_1\tau)+\frac{\tau R_1\sigma_1}{2E_1E_3}\sin(\omega_1\tau)-\frac{\tau R_1\sigma_1}{2E_1^2E_3}\sin(\omega_1\tau)+\frac{\tau^2R_1^2\sigma_1}{4E_1^2E_3}\sin(\omega_1\tau)\sin\gamma_{10}+\frac{\tau R_1^2}{4E_1^2E_3}(\sin(\omega_1\tau))\wedge 2$$

$$\Gamma_{12}=\frac{f}{2\omega_1E_1E_3}\cos\gamma_{10}+\frac{\tau R_1 f}{4\omega_1E_1^2E_3}\sin(\omega_1\tau)\sin\gamma_{10},\Gamma_{13}=\frac{\omega_2^2h}{\omega_1E_1E_3}a_{20}\sin\gamma_{20}+\frac{\tau\omega_2^2hR_1}{2\omega_1E_1^2E_3}a_{20}\sin(\omega_1\tau)\cos\gamma_{20}$$

$$\Gamma_{14}=\frac{\omega_2^2h}{2\omega_1E_1E_3}a_{20}^2\cos\gamma_{20}-\frac{\tau\omega_2^2R_1h}{4\omega_1E_1^2E_3}a_{20}^2\sin(\omega_1\tau)\sin\gamma_{20}$$

$$\Gamma_{21}=-\frac{\sigma_1}{E_1a_{10}}-\frac{\tau R_1\sigma_1}{2E_1a_{10}}\cos(\omega_1\tau)-\frac{R_1}{2a_{10}}\sin(\omega_1\tau)-\frac{\omega_1\varsigma_1E_4}{a_{10}E_1E_3}-\frac{\tau R_1E_4}{2E_1E_3a_{10}}\cos(\omega_1\tau)+\frac{\tau R_1E_4\sigma_1}{2E_1E_3a_{10}}\sin(\omega_1\tau)$$

$$-\frac{\tau R_1E_4\sigma_1}{2E_1^2E_3a_{10}}\sin(\omega_1\tau)+\frac{\tau^2R_1^2E_4\sigma_1}{4E_1^2E_3a_{10}}\sin(\omega_1\tau)\cos(\omega_1\tau)+\frac{\tau R_1^2E_4}{2E_1^2E_3a_{10}}(\sin(\omega_1\tau))\wedge 2$$

$$\Gamma_{22}=-\frac{f}{2\omega_1a_{10}}\sin\gamma_{10}+\frac{E_4 f}{2\omega_1E_1E_3a_{10}}\cos\gamma_{10}+\frac{\tau R_1E_4 f}{4\omega_1E_1^2E_3a_{10}}\sin(\omega_1\tau)\sin\gamma_{10}$$

$$\Gamma_{23}=-\frac{\omega_2^2 h}{\omega_1 a_{10} E_1} a_{20}\cos\gamma_{20}+\frac{\omega_2^2 E_4 h}{\omega_1 E_1 E_3 a_{10}} a_{20}\sin\gamma_{10}+\frac{\tau\omega_2^2 R_1 E_4 h}{2\omega_1 E_1^2 E_3 a_{10}} a_{20}\sin(\omega_1\tau)\cos\gamma_{20}$$

$$\Gamma_{24}=\frac{\omega_2^2 h}{\omega_1 a_{10} E_1} a_{20}^2\cos\gamma_{20}+\frac{\omega_2^2 E_4 h}{\omega_1 E_1 E_3 a_{10}} a_{20}^2\cos\gamma_{20}-\frac{\tau\omega_2^2 R_1 E_4 h}{4\omega_1 E_1^2 E_3 a_{10}} a_{20}^2\sin(\omega_1\tau)\sin\gamma_{20}$$

$$\Gamma_{31}=-\frac{\omega_1^2 g}{4\omega_2 E_2 E_5} a_{20}\sin\gamma_{20}+\frac{\tau\omega_1^2 R_2 g}{16\omega_2^2 E_2^2 E_5}\sin(\omega_2\tau)\cos\gamma_{20}$$

$$\Gamma_{32}=-\frac{\omega_2\varsigma_2}{E_2 E_5}-\frac{R_2}{2E_2E_5}\cos(\omega_2\tau)-\frac{\omega_1^2 g}{4\omega_2 E_2 E_5} a_{10}\sin\gamma_{20}+\frac{\tau R_2^2}{8E_2^2 E_5}(\sin(\omega_2\tau))\wedge 2+\frac{\tau\omega_1^2 R_2 g}{16\omega_2 E_2^2 E_5} a_{10}\sin(\omega_2\tau)\cos\gamma_{20}$$

$$\Gamma_{33}=-\frac{\omega_1^2 g}{4\omega_2 E_2 E_5} a_{10}a_{20}\cos\gamma_{20}-\frac{\tau\omega_1^2 R_2 g}{16\omega_2 E_2^2 E_5} a_{10}a_{20}\sin(\omega_2\tau)\cos\gamma_{20}$$

$$\Gamma_{41}=-\frac{\omega_1^2 g}{4\omega_2}\cos\gamma_{20}-\frac{\omega_1^2 E_6 g}{4\omega_2 E_2^2 E_5}\sin\gamma_{20}+\frac{\tau g\omega_1^2 R_2 E_6}{16\omega_2^2 E_2^2 E_5}\cos\gamma_{20}\sin(\omega_2\tau)-\frac{\sigma_1}{E_1 a_{10}}+\frac{\tau R_1\sigma_1}{2E_1 a_{10}}\cos(\omega_1\tau)+\frac{R_1}{2a_{10}}\sin(\omega_1\tau)$$

$$\frac{\omega_1\varsigma_1 E_4}{E_1 E_3 a_{10}}+\frac{R_1 E_4}{2E_1E_3a_{10}}\cos(\omega_1\tau)-\frac{\tau R_1 E_4\sigma_1}{2E_1E_3a_{10}}\sin(\omega_1\tau)+\frac{\tau R_1 E_4\sigma_1}{2E_1^2E_3a_{10}}\sin(\omega_1\tau)-\frac{\tau^2 R_1^2 E_4\sigma_1}{4E_1^2E_3a_{10}}\sin(\omega_1\tau)\cos(\omega_1\tau)-\frac{\tau R_1^2 E_4}{2E_1^2E_3a_{10}}(\sin(\omega_1\tau))\wedge 2$$

$$\Gamma_{42}=\frac{f}{2\omega_1}\sin\gamma_{10}-\frac{E_4 f}{2\omega_1 E_1 E_3 a_{10}}\cos\gamma_{10}-\frac{\tau R_1 E_4 f}{4\omega_1 E_1^2 E_3 a_{10}}\sin(\omega_1\tau)\sin\gamma_{10}$$

$$\Gamma_{43}=\frac{\sigma_1+\sigma_2}{a_{20}}-\frac{R_2}{2E_2a_{20}}\sin(\omega_2\tau)-\frac{\omega_1^2 g}{4\omega_2E_2a_{20}}a_{10}\cos\gamma_{20}-\frac{\omega_2\varsigma_2E_6}{E_2^2E_5a_{20}}-\frac{R_2E_6}{2E_2^2E_5a_{20}}\cos(\omega_2\tau)-\frac{\omega_1^2 gE_6}{4\omega_2E_2^2E_5a_{20}}a_{10}\sin\gamma_{20}$$

$$+\frac{\tau R_2^2E_6}{8E_2^2E_5a_{20}}(\sin(\omega_1\tau))\wedge 2+\frac{\tau\omega_1^2 gR_2E_6}{16\omega_2E_2^3E_5a_{20}}a_{10}\sin(\omega_1\tau)\cos\gamma_{20}+\frac{\omega_2^2 h}{\omega_1E_1a_{10}}a_{20}\cos\gamma_{20}-\frac{\omega_2^2 hE_4}{\omega_1E_1E_3a_{10}}a_{20}\sin\gamma_{20}$$

$$-\frac{\tau\omega_2^2 hR_1E_4}{2\omega_1E_1^2E_3a_{10}}a_{20}\sin(\omega_1\tau)\cos\gamma_{20}$$

$$\Gamma_{44}=\frac{\omega_1^2 g}{4\omega_2E_2}a_{10}\sin\gamma_{20}-\frac{\omega_1^2 gE_6}{4\omega_2E_2^2E_5}a_{10}\cos\gamma_{20}-\frac{\tau\omega_1^2 gR_2E_4}{16\omega_2E_2^3E_5}a_{20}\sin(\omega_2\tau)\cos\gamma_{20}-\frac{\omega_2^2 h}{2\omega_1E_1a_{10}}a_{20}^2\sin\gamma_{20}$$

$$-\frac{\omega_2^2 hE_4}{2\omega_1E_1E_3a_{10}}a_{20}^2\cos\gamma_{20}+\frac{\tau\omega_2^2 hR_1E_4}{4\omega_1E_1^2E_3a_{10}}a_{20}^2\sin(\omega_1\tau)\sin\gamma_{20}$$

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