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Topological methodologies for generalized multi-granulation

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***ABSTRACT* : Technological revolutions, the rapid pace of life, and the rapid development of approximate theory have led to the need for improved methodologies to deal with huge amounts of data in a short period and give results that are more accurate. This article establishes a model for Multi-Granular Rough Sets (MGRS) within neighborhood systems from a topological perspective. The approach involves approximating a target concept using the j-neighborhoods of objects within a given universe set. We propose extending the multi-granular rough sets framework by employing families of binary relations and constructing multi-topological spaces derived from these multi-relations. This method aims to enhance interior structures and minimize closures. Furthermore, the article examines the properties of these novel methodologies and compares them with previous research. It introduces the concept of a topological membership function in relation to j-neighborhoods of m-topologies, which integrates the principles of rough and fuzzy sets. Our findings indicate that the topology generated from multi-granulation offers more precise accuracy measurements compared to conventional topological methods. Finally, the article presents a real-world application involving medical records to demonstrate the effectiveness of our MGRS classification approach.**

KEYWORDS: Topological operators, N_j-neighborhood systems, Multi-granulation, Rough sets, Approximation structures, Membership function.

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I. INTRODUCTION

Given the rapid growth of data, the challenge of acquiring relevant information for effective decision making has become increasingly urgent. While many scientists have addressed various data-sharing issues with effective solutions, there remains a lack of a cohesive conceptual framework to unify these approaches. Some researchers have developed hybrid methods that integrate mathematics, computer science, and statistics, while others have relied on traditional mathematical techniques. To advance the field, it is essential for researchers to collaborate and harmonize their research frameworks. In the eighties of the last Century Z. Pawlak [11] introduced the theory of rough sets, a foundational mathematical tool for addressing ambiguity, imprecise information, and uncertainty. Pawlak defined an approximation space as an ordered pair (ϒ, R), where R is an equivalence relation on a non-empty finite set (universe) ϒ. The knowledge base in this framework consists of the equivalence classes of ϒ. Pawlak's methodology uses lower and upper approximation operators, defined by these equivalence classes, to categorize data into positive, negative, and boundary regions. However, the constraints of equivalence relations are insufficient for capturing the complex relationships among objects in various real-world domains such as computer networks, economics, medical sciences, and engineering. This limitation affects the efficacy of Pawlak's rough set theory in these contexts. To address these challenges, various extensions of rough set theory have been proposed, including generalized approximation spaces defined by non-equivalence relations. Notably, Yao [16] pioneered the development of the first generalized rough set model based on non-equivalence relations. Instead of equivalence classes, Yao [16] introduced the concepts of "right neighborhood" (^{r}N) and "left neighborhood" (^{l}N) for each object under an arbitrary relation, which serve as granules for approximating information from subsets of data. Subsequent research has led to the creation of various generalized approximation spaces using specific relations such as tolerance reflexive, similarity, and dominance. These developments include generalizations to right and left neighborhoods [10, 15] and the j-neighborhood space based on distinct types of neighborhoods derived from binary relations [2].

Granular computing models offer innovative solutions to challenges in data mining, pattern recognition, and related fields. Nevertheless, certain challenges persist, necessitating further advancements. In 2006, Qian [13] introduced the concept of multi-granular computing, which extends beyond traditional single-granular approaches by incorporating rough set theory. This methodology involves substituting a single relation with a set of relations applied to the same universe, thereby enhancing the flexibility and applicability of granular computing models.

Topology is an effective method for representing relationships between attributes, particularly when addressing complex interactions. Pawlak highlighted the strong connection between topology and rough set theory, asserting that the topological structure of rough sets is a fundamental aspect of the theory. This significant correspondence has spurred researchers to investigate its properties and applications in real-world contexts [1, 3-5, 12]. From a topological perspective, Lin et al. [9] explored a theory of multi-granulation rough sets by deriving multi-topologies from multi-equivalence relations on ϒ. Their research also focused on the topological characteristics of the multi-granulation rough space.

In the present study, we introduced a hybrid methodology that effectively combines topology and rough set theory to tackle the complexities of sharing multi-source, variable, and large-scale data. Specifically, we proposed a topological membership function that integrates the principles of rough and fuzzy sets within the framework of j-neighborhoods of m-topologies. Our findings indicate that topology generated through multi-granulation provides more precise accuracy measurements compared to traditional topological methods.

This article is organized as follows: **Section 2** presents the fundamental concepts and properties of general topology, as well as essential aspects of information systems. **Section 3** establishes topological approaches for generalized multi-granulation using various types of neighborhoods, focusing on the minimization of boundary regions. In **Section 4**, we introduce a medical application from our topological perspective. The article concludes with a summary of key findings and offers insights into potential directions for future research.

II. Preliminaries

The development of rough set theory is driven by the need to describe subsets of a universe in terms of equivalence classes defined by a partition of that universe. This partition forms a topological space known as the approximation space (ϒ, *R*), where ϒ represents the universe and *R* signifies an equivalence relation [11]. The equivalence classes of *R*, also referred to as granules, are represented as $[d]_R$, indicating the equivalence class containing d. Within this approximation space, the lower and upper approximations of any set D are defined as follows:

$$\underline{L}(D) = \{d: [d]_R \subseteq D\},$$
$$\overline{U}(D) = \{d: [d]_R \cap D \neq \phi\},$$

Positive, negative, and boundary regions are represented as:

$$Pos_R(D) = \underline{L}(D),$$
$$Neg_R(D) = \Upsilon \setminus \overline{U}(D),$$
$$Bd_R(D) = \overline{U}(D) \setminus \underline{L}(D).$$

These concepts can be also signified by rough membership functions [8], precisely,

$$M_D^R(d) = \frac{|[d]_R \cap D|}{|[d]_R|}, d \in \Upsilon.$$

various values identifies positive ($M_D^R(d) = 1$), negative ($M_D^R(d) = 0$), and boundary ($0 < M_D^R(d) < 1$) regions. The membership function can be regarded as a form of conditional probability, with its value representing the degree of certainty regarding to a point d ∈ ϒ.

Definition 2.1. [7] A topological space is a pair (ϒ,T) consisting of a set ϒ and a class T of subsets of ϒ satisfying that T is closed under arbitrary union and finite intersection. The family T is a topology on ϒ. The subsets of ϒ, that belong to T are called open sets and their complements are called closed sets.

Definition 2.2. [7] Let (ϒ,T) be topological space and D ⊆ ϒ. Then, the T-interior of D is defined as follows: T-*int*(D) = ⋃{G ⊆ ϒ : G ⊆ D and G is open set}, and the
T-closure of D is defined as follows: T-*cl*(D) = ⋂{F ⊆ ϒ : D ⊆ F and F is closed set}.

Lin [10] introduced the concept of right-neighborhoods to extend the notion of equivalence classes. Various neighborhood systems have since been developed, leading to the creation of numerous generalized approximation spaces. These include generalizations to left neighborhoods [15] and j-neighborhood spaces, which are based on distinct types of neighborhoods derived from binary relations [2].

There are several methods for deriving a topology from a given relation. One such method, employed by Lashin et al. [8], utilizes a topology generated by the class of right-neighborhoods (${}^{r}N(d)$, for any d ∈ ϒ) as a subbase. In their approach, the lower and upper approximation operators are defined using the interior and closure operators of this topology. Additionally, the authors in [8] extend the concept of the rough membership function to topological spaces. If T represents a topology on a universe ϒ, with β as its base, then the rough membership function is defined as:

$$\mathrm{M}_D^{\mathrm{T}}(d) = \frac{|\{\cap \mathrm{B_d}\} \cap D|}{|\{\cap \mathrm{B_d}\}|},\ \mathrm{d} \in \Upsilon$$

where B_d is any member of a base β containing d.

Theorem 2.1. [7] Let (ϒ, T) be a topological space, D ⊆ ϒ then d ∈ T-cl(D) if and only if G⋂D ≠ ∅, for all G ∈ T and d ∈ G.

Theorem 2.2. [15] If ς is a subbase for topologies T, T* on a set ϒ, then T = T*.

Theorem 2.3. [15] Let T, T* be topologies on ϒ generated respectively by subbases ς and ς*. If ς ⊆ ς*, then T ⊆ T*.

Neighborhood structures provide insights into the relationships among objects within a universe. Essentially, objects within a neighborhood exhibit a degree of similarity to a central reference element. The concept of multi-granulation involves employing multiple relations rather than a single one, with the goal of achieving a more nuanced approximation. In the following definition, we explore the j-neighborhoods of a given point, which are derived from a set of m-binary relations.

Definition 2.3. Let R_ℓ, ℓ ∈ {1, 2, ..., m} be m-binary relations on ϒ and *j* ∈ {r, l, i, ũ}. For each ℓ, the *j*-neighborhoods of d ∈ ϒ with respect to R_ℓ (symbolized by ${}^{j}\mathrm{N}_\ell(\mathrm{d})$) are defined as:

1. rℓ-neighborhood of d [16] : ${}^{r}\mathrm{N}_\ell(\mathrm{d}) = \{y \in \Upsilon : \mathrm{d}R_\ell y$, for each $\ell\}$, i.e ${}^{r}\mathrm{N}_\ell(\mathrm{d}) = \bigcap_{\ell=1}^{m}\ {}^{r}\mathrm{N}_{R\ell}(\mathrm{d})$.
2. lℓ-neighborhood of d [16]: ${}^{l}\mathrm{N}_\ell(\mathrm{d}) = \{y \in \Upsilon : yR_\ell\mathrm{d}$, for each $\ell\}$, i.e ${}^{l}\mathrm{N}_\ell(\mathrm{d}) = \bigcap_{\ell=1}^{m}\ {}^{l}\mathrm{N}_{R\ell}(\mathrm{d})$.
3. iℓ-neighborhood of d [2]: ${}^{i}\mathrm{N}_\ell(\mathrm{d}) = {}^{r}\mathrm{N}_\ell(\mathrm{d}) \cap {}^{l}\mathrm{N}_\ell(\mathrm{d})$.
4. ũℓ-neighborhood of d [2]: ${}^{ũ}\mathrm{N}_\ell(\mathrm{d}) = {}^{r}\mathrm{N}_\ell(\mathrm{d}) \cup {}^{l}\mathrm{N}_\ell(\mathrm{d})$.

To generalize multi-granular rough sets, Hussein et. al [6] suggested the generalized multi-interior (briefly, GM-int_r) and generalized multi-closure (briefly, GM-cl_r) operators with respect to right neighborhoods only generated by m-relations.

Definition 2.4. [6] Consider m-topological spaces denoted as $(\Upsilon, \mathrm{T}_\ell)$, ℓ ∈ {1, 2, ...m}, induced by binary relations R_ℓ, ℓ ∈ {1, 2, ...m}. If D ⊆ ϒ, then the GM-int_r and GM-cl_r operators of a set D with respect to the right neighborhoods of the topologies { T_ℓ : ℓ = 1, 2, ...m}, are defined as follows:

$$\text{GM-}int_r(\mathrm{D}) = \bigcup_{\ell=1}^{m}\ \mathrm{T}_\ell\text{-}\ int_r(\mathrm{D}),$$
$$\text{GM-}cl_r(\mathrm{D}) = \bigcap_{\ell=1}^{m}\ \mathrm{T}_\ell\text{-}\ cl_r(\mathrm{D}).$$

The region of topological boundary (GM-bd_r) of a set D is determine by:

$$\text{GM-bd}_r(\mathrm{D}) = \text{GM-}\ cl_r(\mathrm{D}) \setminus \text{GM-}\ int_r(\mathrm{D}).$$

According to Hussein et. al approach [6], the rough membership function is specified as:

Definition 2.5. [6] Consider m-topological spaces denoted as $(\Upsilon, \mathrm{T}_\ell)$, ℓ ∈ {1, 2, ...m}, induced by binary relations R_ℓ, ℓ ∈ {1, 2, ...m} and D ⊆ ϒ. With respect to the right-neighborhoods of the topologies *Γ*= {T_ℓ : ℓ = 1, 2, ...m}, the rough membership function of a point d, is defined as:

$$\mathrm{M}_D^{\Gamma}(d) = \begin{cases} 1 & : \max\limits_{1\le\ell\le m}(\mathrm{M}_D^{\mathrm{T}_\ell}\ (d)) = 1, \\ 0 & : \min_{1\le\ell\le m}(\mathrm{M}_D^{\mathrm{T}_\ell}\ (d)) = 0, \\ \max_{1\le\ell\le m}(\mathrm{M}_D^{\mathrm{T}_\ell}\ (d)) & : otherwise. \end{cases}$$

III. MATERIALS AND METHODS

In this section, a framework for multi-granulation rough sets (MGRS) will be presented, considering a topological spaces viewpoint. The proposed approximations are based multi-topologies from multi-neighborhoods, that induced from multi-relations.

Definition 3.1. Let $(\Upsilon, \mathrm{T}_\ell)$, ℓ ∈ {1, 2, ...m} be m-topological spaces induced by binary relations R_ℓ, ℓ ∈ {1, 2, ...m} and D ⊆ ϒ. If *j* ∈ {r, l, i, ũ}, then the GM-int_j and GM-cl_j operators (or generalized multi-lower approximation and generalized multi-upper approximation, respectively) of a set D with respect to the j-neighborhoods of the topologies $\{T_\ell : \ell = 1, 2, ..., m\}$, are defined as follows:

$$\text{GM-}int_j(D) = \bigcup_{\ell=1}^{m} T_\ell^j\text{-}int(D),$$

$$\text{GM-}cl_j(D) = \bigcap_{\ell=1}^{m} T_\ell^j\text{-}cl(D).$$

The pair (GM-int_j (D), GM-cl_j (D)) is named a MGRS of D.

The region of topological boundary (GM-bd_j) of a set D is determined by:

$$\text{GM-}bd_j(D) = \text{GM-}cl_j(D) \setminus \text{GM-}int_j(D).$$

The measure of topological accuracy (GM-A_j) of a set D is determined by:

$$\text{GM-}A_j(D) = \frac{|\text{GM}-int_j\ (D)|}{|\text{GM}-cl_j\ (D)|},\ |\text{GM-}cl_j(D)| \neq 0.$$

Lemma 3.1. Let (Υ,T_ℓ), $\ell \in \{1, 2, ... , m\}$ be m-topological spaces induced by binary relations R_ℓ, $\ell \in \{1, 2, ..., m\}$ and $D \subseteq \Upsilon$. If $j \in \{r, l, i, \tilde{u}\}$, then

1. GM-int_j (D^c) = (GM-cl_j $(D))^c$,
2. GM-cl_j (D^c) = (GM-int_j $(D))^c$.

Proof: Only the first element will be proved while the second element is proved similarly. Suppose $j \in \{r, l, i, \tilde{u}\}$.

1. GM-int_j $(D^c) = \bigcup_{\ell=1}^{m} T_\ell^j\text{-}int\ (D^c) = \bigcup_{\ell=1}^{m} (T_\ell^j\text{-}cl(D))^c = (\bigcap_{\ell=1}^{m} T_\ell^j\text{-}cl\ (D))^c$ = (GM-cl_j $(D))^c$.

Lemma 3.2. Let (Υ, T_ℓ), $\ell \in \{1, 2, ...m\}$ be m-topological spaces induced by binary relations R_ℓ, $\ell \in \{1, 2, ...m\}$ and $D \subseteq \Upsilon$. If $j \in \{r, l, i, \tilde{u}\}$, then:

1. $T_\ell^j\text{-}int(\text{GM-}int_j(D)) = T_\ell^j\text{-}int(D)$,
2. $T_\ell^j\text{-}cl(\text{GM-}cl_j(D)) = T_\ell^j\text{-}cl(D)$.

Proof: 1. Let j= r. Suppose that $T_\ell^r\text{-}int(D) = S_\ell$, then S_ℓ is the greatest T_ℓ-open set contained in D for each $\ell \in \{1, 2, ... , m\}$. From Definition 3.1., GM-$int_r(D) = \bigcup_{\ell=1}^{m} S_\ell$. Since $\bigcup_{\ell=1}^{m} S_\ell$ is containing S_ℓ, hence $T_\ell^r\text{-}int(\text{GM-}\ int_r(D)) = T_\ell^r\text{-}int(\bigcup_{\ell=1}^{m} S_\ell) = S_\ell$.

2. According to Lemma 3.1., the remaining item will be proved.

Lemma 3.3. Let (Υ, T_ℓ), $\ell \in \{1, 2, ..., m\}$ be m-topological spaces induced by binary relations R_ℓ, $\ell \in \{1, 2, ...m\}$ and $D \subseteq \Upsilon$. If $j \in \{r, l, i, \tilde{u}\}$, then

1. $T_\ell^j\text{-}int(D) \subseteq \text{GM-}int_j(D)$,
2. $\text{GM-}cl_j(D) \subseteq T_\ell^j\text{-}cl(D)$.

Proof: Since GM-$int_j(D) = \bigcup_{\ell=1}^{m} T_\ell^j\text{-}int(D)$, and GM-$cl_j(D) = \bigcap_{\ell=1}^{m} T_\ell^j\text{-}cl(D)$, then by applying Definition 3.1, $T_\ell^j\text{-}int(D) \subseteq \text{GM-}int_j(D)$, and $\text{GM-}cl_j(D) \subseteq T_\ell^j\text{-}cl(D)$ for each $\ell \in \{1, 2, ...m\}$.

Proposition 3.1. Let (Υ, T_ℓ), $\ell \in \{1, 2, ..., m\}$ be m-topological spaces induced by binary relations R_ℓ, $\ell \in \{1, 2, ...m\}$, and $D, \breve{D} \subseteq \Upsilon$. If $j \in \{r, l, i, \tilde{u}\}$, then in the context of the operators GM-int_j and GM-cl_j , one can get the following results:

1. GM-$int_j(\Upsilon) = \Upsilon$, and GM-$int_j(\phi) = \phi$,
2. GM-$cl_j(\Upsilon) = \Upsilon$, and GM-$cl_j(\phi) = \phi$,
3. GM-$int_j(D) \subseteq D \subseteq$ GM-$cl_j(D)$,
4. $D \subseteq \breve{D} \Rightarrow$ GM-$int_j(D) \subseteq$ GM-$int_j(\breve{D})$, GM-$cl_j(D) \subseteq$ GM-$cl_j(\breve{D})$,
5. GM-int_j(GM-$int_j(D)$) = GM-$int_j(D)$, and GM-cl_j(GM-$cl_j(D)$) = GM-$cl_j(D)$.
6. GM-$int_j(D \cap \breve{D})$ = GM-$int_j(D) \cap$ GM-$int_j(\breve{D})$.
7. GM-$cl_j(D \cup \breve{D})$ = GM-$cl_j(D) \cup$ GM-$cl_j(\breve{D})$.

Proof: The proof of the first three items come directly from Definition 3.1. Suppose $j \in \{r, l, i, \tilde{u}\}$.

4. Let $D \subseteq \breve{D}$, then GM-$int_j(D) = \bigcup_{\ell=1}^{m} T_\ell^j\text{-}int(D) \subseteq \bigcup_{\ell=1}^{m} T_\ell^j\text{-}int(\breve{D})$ = GM-$int_j(\breve{D})$, and

$$\text{GM-}cl_j(D) = \bigcap_{\ell=1}^{m} T_\ell^j\text{-}cl\ (D) \subseteq \bigcap_{\ell=1}^{m} T_\ell^j\text{-}cl(\breve{D}) = \text{GM-}cl_j(\breve{D}).$$

5. According to Lemma 3.2.,

$$\text{GM-}int_j\ (\text{GM-}int_j(D)) = (\bigcup_{\ell=1}^{m} T_\ell^j\text{-}int\ (\text{GM-}int_j(D)) = (\bigcup_{\ell=1}^{m} T_\ell^j\text{-}int\ (D)) = \text{GM-}int_j(D)$$

$$\text{GM-}cl_j(\text{GM-}cl_j(D)) = (\bigcap_{\ell=1}^{m} T_\ell^j\text{-}cl\ (\text{GM-}cl_j(D)) = (\bigcap_{\ell=1}^{m} T_\ell^j\text{-}cl\ (D)) = \text{GM-}cl_j(D).$$

6. GM-$int_j(D \cap \breve{D}) = \bigcup_{\ell=1}^{m} T_\ell^j\text{-}int\ (D \cap \breve{D}) = \bigcup_{\ell=1}^{m} T_\ell^j\text{-}int\ (D) \cap \bigcup_{\ell=1}^{m} T_\ell^j\text{-}int\ (\breve{D})$
= GM-$int_j(D) \cap$ GM-$int_j(\breve{D})$.

7. Straightforward.

Corollary 3.1. Let (Υ, T_ℓ), $\ell \in \{1, 2, ..., m\}$ be m-topological spaces induced by binary relations R_ℓ, $\ell \in \{1, 2, ..., m\}$, and D, $D^{\sim} \subseteq \Upsilon$. If $j \in \{r, l, i, \tilde{u}\}$, then GM-$int_j$ and GM-cl_j are interior and closure operators, respectively.

Theorem 3.1. Let (Υ, T_ℓ), $\ell \in \{1, 2, ...,m\}$ be m-topological spaces induced by binary relations R_ℓ, $\ell \in \{1, 2, ..., m\}$. If $j \in \{r, l, i, \tilde{u}\}$, then ${}_{GM}T^j = \{D \subseteq \Upsilon : \text{GM-}int_j(D) = D\}$ is a topology on Υ.

Proof: By applying 1., 2., 3., 5., 6. and 7. for Propositions 3.1., the proof is easy.

Corollary 3.2. For each $j \in \{r, l, i, \tilde{u}\}$, the topology ${}_{GM}T^j$ generated by GM-int_j operator is finer than the topology T_ℓ^j , $\ell \in \{1, 2, ..., m\}$.

Proof: Suppose that $j \in \{r, l, i, \tilde{u}\}$. **Let D** $\subseteq \Upsilon$, then from Lemma 3.3, T_ℓ^j-int(D) $\subseteq$ GM-int_j(D) for each $\ell \in \{1, 2, ...,m\}$. Which implies that $T_\ell^j \subseteq {}_{GM}T^j$ for each $\ell \in \{1, 2, ...,m\}$.

Corollary 3.3. 1. ${}_{GM}T^i \subseteq {}_{GM}T^r \subseteq {}_{GM}T^{\tilde{u}}$,

2. ${}_{GM}T^i \subseteq {}_{GM}T^l \subseteq {}_{GM}T^{\tilde{u}}$.

Proof: Straightforward.

According to $\bigcup_{\ell=1}^{m} T_\ell^j$ is not a topology on Υ, the next remark is understandable.

Remark 3.1. For each $j \in \{r, l, i, \tilde{u}\}$, ${}_{GM}T^j \neq \bigcup_{\ell=1}^{m} T_\ell^j$.

Theorem 3.2. Let (Υ, T_ℓ), $\ell \in \{1, 2, ..., m\}$ be m-topological spaces induced by binary relations R_ℓ, $\ell \in \{1, 2, ..., m\}$, and D, $D^{\sim} \subseteq \Upsilon$. Then

1. GM-int_i(D) $\subseteq$ GM-int_r(D) $\subseteq$ GM-$int_{\tilde{u}}$ (D) $\subseteq$ D $\subseteq$ GM-$cl_{\tilde{u}}$(D) $\subseteq$ GM-cl_r(D) $\subseteq$ GM-cl_i(D).
2. GM-int_i(D) $\subseteq$ GM-int_l(D) $\subseteq$ GM-$int_{\tilde{u}}$ (D) $\subseteq$ D $\subseteq$ GM-$cl_{\tilde{u}}$(D) $\subseteq$ GM-cl_l(D) $\subseteq$ GM-cl_i(D).

Corollary 3.4. 1. GM-A_i(D) $\leq$ GM-A_r(D) $\leq$ GM-$A_{\tilde{u}}$ (D).

2. GM-A_i(D) $\leq$ GM-A_l(D) $\leq$ GM-$A_{\tilde{u}}$ (D).

The following example demonstrates that the inclusions in Theorem 3.2 and Corollaries 3.3 and 3.4 cannot be substituted with equality.

Example 3.1. Suppose that $\Upsilon = \{1, 2, 3, 4, 5\}$, D = {1, 2, 4}. Let R_1 = {(1, 2), (1, 3), (2, 4), (2, 5), (5, 1)}, R_2 = {(2, 2), (3, 4), (4, 1), (4, 5), (5, 3)}, and R_3 = {(1, 1), (3, 4), (3, 2), (4, 1), (5, 2), (5, 3)} be binary relations on a universe Υ. The topologies induced from the rℓ-neighborhoods, lℓ-neighborhoods, iℓ-neighborhoods, ũℓ-neighborhoods of the above relations are presented as follows:

$T_1^r = \{\phi, \{1\}, \{2, 3\}, \{4, 5\}, \{1, 2, 3\}, \{1, 4, 5\}, \{2, 3, 4, 5\}, \Upsilon\}$,

$T_2^r = \{\phi, \{2\}, \{3\}, \{4\}, \{1, 5\}, \{2, 4\}, \{2, 3\}, \{3, 4\}, \{1, 3, 5\}, \{1, 2, 5\}, \{1, 4, 5\}, \{1, 2, 4, 5\}, \{2, 3, 4\}, \{1, 2, 3, 5\}, \{1, 3, 4, 5\}, \Upsilon\}$,

$T_3^r = \{\phi, \{1\}, \{2\}, \{2, 4\}, \{2, 3\}, \{1, 2, 4\}, \{1, 2, 3\}, \{2, 3, 4\}, \{1, 2\}, \{1, 2, 3, 4\}, \Upsilon\}$.

${}_{GM}T^r = P(\Upsilon) \setminus \{\{5\}, \{2, 5\}, \{3, 5\}, \{2, 3, 5\}\}$, where P($\Upsilon$) is the power set of universe Υ.

Table 1 presents a comparison of the accuracy achieved by each topology and our proposed methodology with respect to a particular data set D.

Table 1: Comparison among accuracy measures of D

Approximation Space	*int*(D)	*cl*(D)	Accuracy
(Υ, T_1^r)	{1}	Υ	0.2
(Υ, T_2^r)	{2, 4}	{1, 2, 4, 5}	0.5
(Υ, T_3^r)	{1, 2, 4}	Υ	0.6
Our methodology	{1, 2, 4}	{1, 2, 4, 5}	0.75

Positive, negative, and boundary regions are represented as:

GM- Pos_r(D) = {1, 2, 4}, GM- Neg_r(D) = {3}, GM-bd_r(D) = {5}.

T_1^{l} = {ϕ, {1}, {2}, {5}, {1, 2}, {1, 5}, {2, 5}, {1, 2, 5}, ϒ},
T_2^{l} = {ϕ, {2}, {3}, {4}, {5}, {2, 3}, {2, 4}, {2, 5}, {3, 4}, {3, 5}, {4, 5}, {2, 3, 4}, {2, 3, 5}, {3, 4, 5}, {2, 4, 5}, {2, 3, 4, 5}, ϒ},
T_3^{l} = {ϕ, {1, 4}, {3, 5}, {5}, {3}, {1, 3, 4}, {1, 4, 5}, {1, 3, 4, 5}, ϒ}.
$_{GM}T^{l}$ = P(ϒ).

Table 2 presents a comparison of the accuracy achieved by each topology and our proposed methodology with respect to a particular data set D.

Table 2: Comparison among accuracy measures of D

Approximation Space	*int*(D)	*cl*(D)	Accuracy
(ϒ, T_1^{l})	{1, 2}	{1, 2, 3, 4}	0.5
(ϒ, T_2^{l})	{2, 4}	{1, 2, 4}	0.6
(ϒ, T_3^{l})	{1, 4}	{1, 2, 4}	0.6
Our methodology	{1, 2, 4}	{1, 2, 4}	1

Positive, negative, and boundary regions are represented as:
GM- Pos_l(D) = {1, 2, 4}, GM- Neg_l(D) = {3, 5}, GM-bd_l(D) = ϕ.

T_1^{i} = {ϕ, ϒ},
T_2^{i} = {ϕ, {2}, ϒ},
T_3^{i} = {ϕ, {1}, ϒ}.
$_{GM}T^{i}$ = { ϕ, {1}, {2}, {1, 2}, ϒ}.

Table 3 presents a comparison of the accuracy achieved by each topology and our proposed methodology with respect to a particular data set D.

Table 3: Comparison among accuracy measures of D

Approximation Space	*int*(D)	*cl*(D)	Accuracy
(ϒ, T_1^{i})	ϕ	ϒ	0
(ϒ, T_2^{i})	{2}	ϒ	0.2
(ϒ, T_3^{i})	{1}	ϒ	0.2
Our methodology	{1,2}	ϒ	0.4

Positive, negative, and boundary regions are represented as:
GM- Pos_i(D) = {1, 2}, GM- Neg_i(D) = ϕ, GM-bd_i(D) = {3, 4, 5}.

$T_1^{\tilde{u}}$ = {ϕ, {1}, {2}, {5}, {1, 2}, {2, 3, 5}, {1, 4, 5}, {1, 2, 3, 5}, {1, 2, 4, 5}, {1, 5}, {2, 5}, {1, 2, 5}, ϒ},
$T_2^{\tilde{u}}$= { ϕ , {2}, {4}, {4, 5}, {3, 4}, {1, 3, 5}, {3}, {5}, {2, 3}, {2, 5}, {3, 5}, {3, 4, 5}, {2, 3, 5}, {2, 4}, {2, 4, 5}, {2, 3, 4}, {1, 2, 3, 5}, {1, 3, 4, 5}, {2, 3, 4, 5}, ϒ},
$T_3^{\tilde{u}}$ = {ϕ, {1, 3}, {1, 4}, {3, 5}, {2, 3}, {2, 4, 5}, {1}, {2}, {3}, {4}, {5}, {1, 3, 4}, {1, 3, 5}, {1, 2, 3}, {1, 3, 4, 5}, {1, 2, 3, 4}, {1, 2, 4, 5}, {2, 3, 5}, {2, 3, 4, 5}, {1, 4, 5}, {3, 4, 5}, {2, 3, 4}, {1, 2, 3, 5}, {1, 2}, {1, 5}, {3, 5}, {3, 4}, {2, 5}, {4, 5}, {1, 2, 4}, {{2, 4}, {1, 2, 5}, ϒ}.
$_{GM}T^{\tilde{u}}$ = P(ϒ).

Table 4 presents a comparison of the accuracy achieved by each topology and our proposed methodology with respect to a particular data set D.

Table 4: Comparison among accuracy measures of D

Approximation Space	*int*(D)	*cl*(D)	Accuracy
(ϒ, $T_1^{\tilde{u}}$)	{1, 2}	{1, 2, 3, 4}	0.5
(ϒ, $T_2^{\tilde{u}}$)	{2, 4}	{1, 2, 4}	0.6
(ϒ, $T_3^{\tilde{u}}$)	{1, 2, 4}	{1, 2, 4}	1
Our methodology	{1, 2, 4}	{1, 2, 4}	1

Positive, negative, and boundary regions are represented as:
GM- $Pos_{\tilde{u}}$(D) = {1, 2, 4}, GM- $Neg_{\tilde{u}}$(D) ={3, 5}, GM-$bd_{\tilde{u}}$(D) = ϕ.

Remark 3.2. According to Example 3.1., it should be noted that:
1. ${}_{GM}T^{i} \subseteq {}_{GM}T^{l}$, although $T^{i}_{\ell} \nsubseteq T^{l}_{\ell}$,
2. ${}_{GM}T^{r} \subseteq {}_{GM}T^{\tilde{u}}$, although $T^{r}_{\ell} \nsubseteq T^{\tilde{u}}_{\ell}$.
3. In the case of the set D = {1, 2, 4} we found that ${}_{GM}T^{l} = {}_{GM}T^{\tilde{u}} < {}_{GM}T^{i}$ and ${}_{GM}T^{r}$.

By the membership function, another technique to calculate positive, negative, and boundary regions are given.
Definition 3.2. Let (Υ, T_ℓ), $\ell \in \{1, 2, ...m\}$ be m-topological spaces induced by binary relations R_ℓ, $\ell \in \{1, 2, ...m\}$ and $d \in \Upsilon$. If $j \in \{r, l, i, \tilde{u}\}$, then a membership function with respect to the j-neighborhoods of the topologies $\{T_\ell : \ell = 1, 2, ..., m\}$, is defined as follows:

$$M^{S}_{D}(d) = \begin{cases} 1 & : \max_{1\le \ell \le m} (M_D^{T^j_\ell}(d)) = 1, \\ 0 & : \min_{1\le \ell \le m} (M_D^{T^j_\ell}(d)) = 0, \\ \max_{1\le \ell \le m} (M_D^{T^j_\ell}(d)) & : otherwise. \end{cases}$$

Where $S = \{T^{j}_{\ell} : \ell = 1, 2, ...m, j \in \{ r, l, i, \tilde{u}\}\}$.

The following example highlights Definition 3.2.
Example 3.2. Let Υ, R_1, R_2 and R_3 be as in Example 3.1. β^{j}_{ℓ}, are the basis of T^{j}_{ℓ}, $\ell = \{1, 2, 3\}$, $j \in \{ r, l, i, \tilde{u}\}$, respectively.
For D = {1, 2, 4} we have:

$$\beta^{r}_{1} = \{\phi, \{2, 3\}, \{4, 5\}, \{1\}, \Upsilon\},$$
$$\beta^{r}_{2} = \{\phi, \{2\}, \{4\}, \{1, 5\}, \{3\}, \Upsilon\},$$
$$\beta^{r}_{3} = \{\phi, \{1\}, \{2\}, \{2, 4\}, \{2, 3\}, \Upsilon\}.$$

$M_D^{T_1^r}(1)= 1$, $M_D^{T_1^r}(2) = \frac{1}{2}$, $M_D^{T_1^r}(3) = \frac{1}{2}$, $M_D^{T_1^r}(4) = \frac{1}{2}$, $M_D^{T_1^r}(5) = \frac{1}{2}$.
$M_D^{T_2^r}(1)= \frac{1}{2}$, $M_D^{T_2^r}(2) = 1$, $M_D^{T_2^r}(3)= 0$, $M_D^{T_2^r}(4)= 1$, $M_D^{T_2^r}(5) = \frac{1}{2}$.
$M_D^{T_3^r}(1)= 1$, $M_D^{T_3^r}(2) = 1$, $M_D^{T_3^r}(3) = \frac{1}{2}$, $M_D^{T_3^r}(4)= 1$, $M_D^{T_3^r}(5) = \frac{3}{5}$.
$M_D^{S}(1)= 1$, $M_D^{S}(2) = 1$, $M_D^{S}(3)= 0$, $M_D^{S}(4)=1$, $M_D^{S}(5) = \frac{3}{5}$.

Then, GM-int_r(D) = {1, 2, 4} and GM-cl_r(D) = {1, 2, 4, 5} which insures the result in Example 3.1., Table 1
Positive, negative, and boundary regions are represented as:
GM- Pos_r(D) = {1, 2, 4}, GM- Neg_r(D) ={3}, GM-bd_r(D) ={5}.

$$\beta^{l}_{1} = \{\phi, \{1\}, \{2\}, \{5\}, \Upsilon\},$$
$$\beta^{l}_{2} = \{\phi, \{2\}, \{3\}, \{4\}, \{5\}, \Upsilon\},$$
$$\beta^{l}_{3} = \{\phi, \{1, 4\}, \{3, 5\}, \{5\}, \{3\}, \Upsilon\},$$

$M_D^{T_1^l}(1)= 1$, $M_D^{T_1^l}(2)= 1$, $M_D^{T_1^l}(3)= \frac{3}{5}$, $M_D^{T_1^l}(4)= \frac{3}{5}$, $M_D^{T_1^l}(5)= 0$.
$M_D^{T_2^l}(1)= \frac{3}{5}$, $M_D^{T_2^l}(2)=1$, $M_D^{T_2^l}(3)= 0$, $M_D^{T_2^l}(4)= 1$, $M_D^{T_2^l}(5)= 0$.
$M_D^{T_3^l}(1)=1$, $M_D^{T_3^l}(2)= \frac{3}{5}$, $M_D^{T_3^l}(3)= 0$, $M_D^{T_3^l}(4)=1$, $M_D^{T_3^l}(5)= 0$.
$M_D^{S}(1)= 1$, $M_D^{S}(2)= 1$, $M_D^{S}(3)= 0$, $M_D^{S}(4)=1$, $M_D^{S}(5)=0$.

Then, GM-int_l(D) = {1, 2, 4} and GM-cl_l(D) = {1, 2, 4} which insures the result in Example 3.1., Table 2. Positive, negative, and boundary regions are represented as:
GM- Pos_l(D) = {1, 2, 4}, GM- Neg_l(D) ={3, 5}, GM-bd_l(D) = ϕ.

$$\beta^{i}_{1} = \{\phi, \Upsilon\},$$
$$\beta^{i}_{2} = \{\phi, \{2\}, \Upsilon\},$$
$$\beta^{i}_{3} = \{\phi, \{1\}, \Upsilon\}.$$

$M_D^{T_1^i}(1)= \frac{3}{5}$, $M_D^{T_1^i}(2)= \frac{3}{5}$, $M_D^{T_1^i}(3)= \frac{3}{5}$, $M_D^{T_1^i}(4)= \frac{3}{5}$, $M_D^{T_1^i}(5)= \frac{3}{5}$.

$M_D^{T_2^{i}}(1)=\frac{3}{5}$, $M_D^{T_2^{i}}(2)=1$, $M_D^{T_2^{i}}(3)=\frac{3}{5}$, $M_D^{T_2^{i}}(4)=\frac{3}{5}$, $M_D^{T_2^{i}}(5)=\frac{3}{5}$.

$M_D^{T_3^{i}}(1)=1$, $M_D^{T_3^{i}}(2)=\frac{3}{5}$, $M_D^{T_3^{i}}(3)=\frac{3}{5}$, $M_D^{T_3^{i}}(4)=\frac{3}{5}$, $M_D^{T_3^{i}}(5)=\frac{3}{5}$.

$M_D^{S}(1)=1$, $M_D^{S}(2)=1$, $M_D^{S}(3)=\frac{3}{5}$, $M_D^{S}(4)=\frac{3}{5}$, $M_D^{S}(5)=\frac{3}{5}$.

Then, GM-int_i(D) = {1, 2} and GM-cl_i(D) = ϒ which insures the result in Example 3.1., Table 3.
Positive, negative, and boundary regions are represented as:

GM- Pos_i(D) = {1, 2}, GM- Neg_i(D) = ϕ, GM-bd_i(D) ={3, 4, 5}.

ß$_1^{\tilde{u}}$ = {ϕ, {1}, {2}, {1, 2}, {2, 3, 5}, {1, 4, 5}, {5}, ϒ},
ß$_2^{\tilde{u}}$ = {ϕ, {2}, {4}, {4, 5}, {1, 3, 5}, {3, 4}, {3}, {5}, ϒ},
ß$_3^{\tilde{u}}$= {ϕ, {1, 4}, {3, 5}, {2, 4, 5}, {1, 3}, {2, 3}, {1}, {2}, {3}, {4}, {5}, ϒ}.

$M_D^{T_1^{\tilde{u}}}(1)=1$, $M_D^{T_1^{\tilde{u}}}(2)=1$, $M_D^{T_1^{\tilde{u}}}(3)=\frac{1}{3}$, $M_D^{T_1^{\tilde{u}}}(4)=\frac{2}{3}$, $M_D^{T_1^{\tilde{u}}}(5)=0$.

$M_D^{T_2^{\tilde{u}}}(1)=\frac{1}{3}$, $M_D^{T_2^{\tilde{u}}}(2)=1$, $M_D^{T_2^{\tilde{u}}}(3)=0$, $M_D^{T_2^{\tilde{u}}}(4)=1$, $M_D^{T_2^{\tilde{u}}}(5)=0$.

$M_D^{T_3^{\tilde{u}}}(1)=1$, $M_D^{T_3^{\tilde{u}}}(2)=1$, $M_D^{T_3^{\tilde{u}}}(3)=0$, $M_D^{T_3^{\tilde{u}}}(4)=1$, $M_D^{T_3^{\tilde{u}}}(5)=0$.

$M_D^{S}(1)=1$, $M_D^{S}(2)=1$, $M_D^{S}(3)=0$, $M_D^{S}(4)=1$, $M_D^{S}(5)=0$.

Then, GM-$int_{\tilde{u}}$(D) = {1, 2, 4} and GM-$cl_{\tilde{u}}$(D) = {1, 2, 4} which insures the result in Example 3.1., Table 4.
Positive, negative, and boundary regions are represented as:

GM- $Pos_{\tilde{u}}$(D) = {1, 2, 4}, GM- $Neg_{\tilde{u}}$(D) = {3, 5}, GM-$bd_{\tilde{u}}$(D) = ϕ.

Remark 3.3. According to Examples 3.1, 3.2 it should be noted that there are various methods for approximating sets utilizing generalized multi-lower approximation GM-int_j and generalized multi-upper approximation GM-cl_j, $j \in$ { r, l, i, ũ}. Among these methods, the most effective is given when j = l or ũ in constructing the approximations of sets, where the boundary regions in these cases are eliminated by increasing the lower approximation and decreasing the upper approximation. Additionally, the GM-accuracy is more accurate than the other types since GM-A_i(D) ≤ GM-A_r(D) ≤ GM-$A_{\tilde{u}}$ (D) and GM-A_i(D) ≤ GM-A_l(D) = GM-$A_{\tilde{u}}$ (D), for any subset D of ϒ.

IV. Real life applications

The prevalence of these lesions in patients with digestive diseases can be attributed to the consumption of processed meat and fast food, both of which are high in calories. This diet often leads to excessive caloric intake and predisposes individuals to future digestive system disorders, including the most severe cases of colon and stomach malignancies. When food bypasses the stomach and enters the intestine directly, it disrupts the absorption process. Post-meal, patients often experience severe symptoms such as headaches, dizziness, colic, and elevated blood sugar levels. Over time, these individuals may develop serious conditions, including high cholesterol and clogged arteries, potentially resulting in heart attacks.
Inherited stomach and colon cancer syndromes primarily include two forms:

- **Hereditary Non-polyposis Colorectal Cancer (HNPCC)**: Also known as Lynch syndrome, HNPCC increases the risk of colon, stomach, and other cancers. Individuals with HNPCC are more likely to develop stomach and colon cancers before the age of fifty.
- **Familial adenomatous polyposis (FAP)**: FAP is rare condition characterized by the development of thousands of polyps in the rectum, colon, and stomach's lining. Individuals with untreated FAP have a significantly increased risk of developing stomach and colon cancer before the age of forty.

The following characteristics match the medical reports that the doctor [6] in this instance sought for the seven patients ϒ = {A, B, C, D, E, F, G}:
1) Liver Functions: of the type S. GPT (**ŁƑ1**) (Normal percent between 0 to 45 U/L) and of the type S. GOT (**ŁƑ2**) (Normal percent between 0 to 37 U/L).
2) Kidney Functions (**ƙƑ**): The measurements of uric acid in the blood (Uric Acid varies between 3 to 7 mg/dl).
3) Fats Percentage: Fats in the blood are divided into two types, the cholesterol level that has a normal range less than 200 mg/dl, the border range is between 200 to 240 mg/dl, the critical range of it that causes arteriosclerosis or heart is higher than 240 mg/dl (**ƑƤ1**). Second, the so-called triglycerides range that has reference up to 150 mg/dl (**ƑƤ2**).

4) Heart Efficiency (**ⱧĘ**): we measured the enzyme (Serum LDH) that has ranged reference between 0 to 480 U/L.
5) Signs of Tumors: we tested the digestive system through the scale (CEA) and normal Non-smoking rooms if less than 5 mg/ml (**ŞŢ1**). The other measure so-called CA 19.9 and extent of reference from 0 to 39 U/ml (**ŞŢ2**).
6) Viruses Hepatitis: Test the patient's immunity against of viruses of type B (HBC) (**ƲⱧ(B))** and of type C (**ƲⱧ(C)**) (Highly infectious) furthermore is positive or negative.
7) Blood Sugar (**ƁŞ**): The patient measurement of sugar of fasting for 6 hours, and an hour after eating, and then two hours after eating.
The results of the seven patients were collected from official files in the physician, which has been done after six months of surgery. We get this data from [6], (see Table 5).

Table 5: Medical Decision Information System

Patients	age	ŁƑ1	ŁƑ2	ƙƑ	ƑƤ1	ƑƤ2	ⱧĘ	ŞŢ1	ŞŢ2	ƲⱧ(B)	ƲⱧ(C)	ƁŞ	Decision
A	12	63	45	11.2	180	210	526	36	44	N	N	N	N
B	5	50	44	4.7	255	188	512	11	26	N	P	N	N
C	18	34.5	23	5.6	177	112	430	16	36	N	N	P	P
D	22	55	33	14.2	311	240	515	28	49	P	P	P	P
E	8	36	22	6.3	166	99	310	11	23	N	N	N	N
F	13	49	50	8.5	230	120	420	18	24	P	N	N	N
G	15	57.5	41	7.6	206	144	460	17	25	N	P	P	P

Where, N means negative and p means positive.
For every attribute, we establish a proper relation, and we use our technique on this data in the following ways:
$R_{\text{age}} = \{(d, \tilde{d}): |f_{\text{age}}(d) - f_{\text{age}}(\tilde{d})| \le 3\}$,
$R_{\text{ŁƑ1}} = \{(d, \tilde{d}): f_{\text{ŁƑ1}}(d) \text{ and } f_{\text{ŁƑ1}}(\tilde{d}) \le 45 \text{ or } f_{\text{ŁƑ1}}(d) \text{ and } f_{\text{ŁƑ1}}(\tilde{d}) > 45\}$,
$R_{\text{ŁƑ2}} = \{(d, \tilde{d}): f_{\text{ŁƑ2}}(d) \text{ and } f_{\text{ŁƑ2}}(\tilde{d}) \le 37 \text{ or } f_{\text{ŁƑ2}}(d) \text{ and } f_{\text{ŁƑ2}}(\tilde{d}) > 37\}$,
$R_{\text{ƙƑ}} = \{(d, \tilde{d}): 3 \le f_{\text{ƙƑ}}(d) \text{ and } f_{\text{ƙƑ}}(\tilde{d}) \le 7, f_{\text{ƙƑ}}(d) \text{ and } f_{\text{ƙƑ}}(\tilde{d}) < 3 \text{ or } f_{\text{ƙƑ}}(d) \text{ and } f_{\text{ƙƑ}}(\tilde{d}) > 7\}$,
$R_{\text{ƑƤ1}} = \{(d, \tilde{d}): 200 \le f_{\text{ƑƤ1}}(d) \text{ and } f_{\text{ƑƤ1}}(\tilde{d}) \le 240, f_{\text{ƑƤ1}}(d) \text{ and } f_{\text{ƑƤ1}}(\tilde{d}) < 200 \text{ or } f_{\text{ƑƤ1}}(d) \text{ and } f_{\text{ƑƤ1}}(\tilde{d}) > 240\}$,
$R_{\text{ƑƤ2}} = \{(d, \tilde{d}): f_{\text{ƑƤ2}}(d) \text{ and } f_{\text{ƑƤ2}}(\tilde{d}) \le 150 \text{ or } f_{\text{ƑƤ2}}(d) \text{ and } f_{\text{ƑƤ2}}(\tilde{d}) > 150\}$,
$R_{\text{ⱧĘ}} = \{(d, \tilde{d}): f_{\text{ⱧĘ}}(d) \text{ and } f_{\text{ⱧĘ}}(\tilde{d}) \le 480 \text{ or } f_{\text{ⱧĘ}}(d) \text{ and } f_{\text{ⱧĘ}}(\tilde{d}) > 480\}$,
$R_{\text{ŞŢ1}} = \{(d, \tilde{d}): f_{\text{ŞŢ1}}(d) \text{ and } f_{\text{ŞŢ1}}(\tilde{d}) \le 5 \text{ or } f_{\text{ŞŢ1}}(d) \text{ and } f_{\text{ŞŢ1}}(\tilde{d}) \le 15 \text{ or } f_{\text{ŞŢ1}}(d) \text{ and } f_{\text{ŞŢ1}}(\tilde{d}) > 15\}$,
$R_{\text{ŞŢ2}} = \{(d, \tilde{d}): f_{\text{ŞŢ2}}(d) \text{ and } f_{\text{ŞŢ2}}(\tilde{d}) \le 39 \text{ or } f_{\text{ŞŢ2}}(d) \text{ and } f_{\text{ŞŢ2}}(\tilde{d}) > 39\}$,
$R_{\text{ƲⱧ(B)}} = \{(d, \tilde{d}): f_{\text{ƲⱧ(B)}}(d) = f_{\text{ƲⱧ(B)}}(\tilde{d})\}$,
$R_{\text{ƲⱧ(C)}} = \{(d, \tilde{d}): f_{\text{ƲⱧ(C)}}(d) = f_{\text{ƲⱧ(C)}}(\tilde{d})\}$,
$R_{\text{ƁŞ}} = \{(d, \tilde{d}): f_{\text{ƁŞ}}(d) = f_{\text{ƁŞ}}(\tilde{d})\}$.
Thus, we compute relations as follows:
R_{age} = {(A, F), (A, G), (B, E), (C, G), (E, B), (F, A), (G, A), (G, C), (G, F), (F, G)}.
$R_{\text{ŁƑ1}}$ = {(C, E), (E, C), (A, B), (A, D), (A, F), (A, G), (B, A), (B, D), (B, F), (B, G), (D, A), (D, B), (D, F), (D, G), (F, A), (F, B), (F, D), (F, G), (G, A), (G, B), (G, D), (G, F)}.
$R_{\text{ŁƑ2}}$ = {(A, B), (A, F), (A, G), (B, A), (B, F), (B, G), (F, A), (F, B), (F, G), (G, A), (G, B), (G, F), {A, B, (C, D), (C, E), (D, C), (D, E), (E, C), (E, D)}.
$R_{\text{ƙƑ}}$ = {(A, D), (A, F), (A, G), (D, A), (D, F), (D, G), (F, A), (F, D), (F, G), (G, A), (G, D), (G, F), (B, C), (B, E), (C,B), (C, E), (E, B), (E, C)}.
$R_{\text{ƑƤ1}}$ = {(A, C), (A, E), (C, A), (C, E), (E, A), (E, C), (B, D), (D, B), (F, G), (G, F)}.
$R_{\text{ƑƤ2}}$ = {(A, B), (A, D), (B, A), (B, D), (D, A), (D, B), (C, E), (C, F), (C, G), (E, C), (E, F), (E, G), (F, C), (F, E), (F, G), (G, C), (G, E), (G, F)}.
$R_{\text{ⱧĘ}}$ = {(A, B), (A, D), (B, A), (B, D), (D, A), (D, B), (C, E), (C, F), (C, G), (E, C), (E, F), (E, G), (G, C), (G, E), (G, F)}.
$R_{\text{ŞŢ1}}$ = {(A, C), (A, D), (A, F), (A, G), (C, A), (C, D), (C, F), (C, G), (D, A), (D, C), (D, F), (D, G), (F, A), (F, C), (F, D), (F, G), (G, A), (G, C), (G, D), (G, F), (B, E), (E, B)}.
$R_{\text{ŞŢ2}}$ = {(A, D), (D, A), (B, C), (B, E), (B, F), (B, G), (C, B), (C, E), (C, F), (C, G), (E, B), (E, C), (E, F), (E, G), (F, B), (F, C), (F, E), (F, G), (G, B), (G, C), (G, E), (G, F)}.
$R_{\text{ƲⱧ(B)}}$ = {(A, B), (A, C), (A, E), (A, G), (B, A), (B, C), (B, E), (B, G), (C, A), (C, B), (C, E), (C, G), (E, A), (E, B), (E, C), (E, G), (G, A), (G, B), (G, C), (G, E), (D, F), (F, D)}.
$R_{\text{ƲⱧ(C)}}$ = {(A, C), (A, E), (A, F), (C, A), (C, E), (C, F), (E, A), (E, C), (E, F), (F, A), (F, C), (F,E), (B, D), (B, G), (D, B), (D, G), (G, B), (G, D)}.
$R_{\text{ƁŞ}}$ = {(A, B), (A, E), (A, F), (B, A), (B, E), (B, F), (E, A), (E, B), (E, F), (F, A), (F, B), (F, E), (C, D), (C, G), (D, C), (D, G), (G, C), (G, D)}.

According to Example 3.1, we compute the topology of every relation and get a next table of comparisons among accuracy measures of a set D~ = {A, B, E, F}

Table 6: Comparison among accuracy measures of D~

	int(D~)	*cl*(D~)	Accuracy
(Υ, T^r_{age})	{A, B, E, F}	{A, B, C, D, E, F}	0.6
$(\Upsilon, T^r_{ŁF1})$	{A, B, E, F}	{A, B, D, E, F, G}	0.57
$(\Upsilon, T^r_{ŁF2})$	{A, B, E, F}	{A, B, E, F, G}	0.8
$(\Upsilon, T^r_{ꞰF})$	{A, B, E, F}	{A, B, E, F}	1
$(\Upsilon, T^r_{ƑP1})$	{A, B, E, F}	{A, B, E, F}	1
$(\Upsilon, T^r_{ƑP2})$	{A, B, E, F}	{A, B, E, F}	1
$(\Upsilon, T^r_{ĦĘ})$	{A, B, E, F}	{A, B, E, F}	1
$(\Upsilon, T^r_{ȘT1})$	{B, E}	{A, B, E, F}	0.5
$(\Upsilon, T^r_{ȘT2})$	{A, B, E, F}	{A, B, C, E, F, G}	0.6
$(\Upsilon, T^r_{VH(B)})$	{A, B, E, F}	{A, B, C, E, F, G}	0.6
$(\Upsilon, T^r_{VH(c)})$	{A, B, E, F}	{A, B, C, E, F}	0.8
$(\Upsilon, T^r_{ƁȘ})$	{A, B, E, F}	{A, B, E, F}	1
Our methodology	{A, B, E, F}	{A, B, E, F}	1

V. Discussion and analysis of results

By observing the inferred relationship, we find that the symmetry relationship is achieved, and this leads to the equality of the relationships when $j \in$ {r, l, i, ũ}. The accuracy of approximating the concept D~ using only the information in the data table is 100%. A preliminary analysis indicates that this accuracy is consistent across the categories {ꞰF, ƑP1, ƑP2, ĦĘ, ƁȘ}. These results suggest that this method is highly effective for practical applications, as it successfully reduces the number of medical tests required for disease diagnosis from 12 to just 5. This reduction facilitates more efficient decision-making for patients.

Hussein et al. [6] proposed the generalized multi-interior (GM-intr) and generalized multi-closure (GM-clr) operators, based on right neighborhoods only that generated by m-relations. To advance the concept of multi-granular rough sets, we build upon the perspective of Hussein et al. [6], introducing the generalized multi-interior (GM-intj) and generalized multi-closure (GM-clj) operators. These operators are based on j-neighborhoods generated by m-relations, where j∈{r, l, i, ũ}.

In contrast to the results reported by Radwan et al. [14] in Section 3, which involved generating topologies using the approach developed by Abd El-Monsef et al. [2], our current work utilizes the technique proposed by Lashin et al. [8], which is based on different types of neighborhoods. Consequently, the methodology employed in our study and that used by Radwan et al. [14] are fundamentally distinct.

VI. Conclusions And Future Works

In this paper, we presented a multi-granulation rough set model that relies on specific types of neighborhood systems (NS). We demonstrated that this new model represents a generalization of existing multi-granulation rough set (MGRS) models from a topological perspective. Specifically, we introduced the concept of a topological membership function in relation to j-neighborhoods of m-topologies, integrating the principles of fuzzy and rough sets. Our research indicates that the topology generated through multi-granulation provides more precise accuracy measurements compared to traditional topological approaches.

In future research, we aim to explore further generalizations of topological concepts, such as near closed and near open sets. We plan to apply these broadly applicable ideas to large-scale, real-world datasets. Additionally, generalized topological concepts will be utilized to investigate multivariate data reduction. To advance these efforts, there is a need for new techniques and tools capable of automatically and intelligently extracting implicit knowledge from data. Furthermore, fostering greater integration between various scientific disciplines will be essential for enhancing our understanding of the world and improving our quality of life.

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