Bulletin of Faculty of Science ,Zagazig University (BFSZU) ***2024***
Bulletin of Faculty of Science, Zagazig University (BFSZU)
e-ISSN: 1110-1555
Volume-2024, Issue-4, pp-57-63
https://bfszu.journals.ekb.eg/journal
Research Paper DOI: **10.21608/bfszu.2024.270134.1365**

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Optimizing Multi-Skill Call Center Performance: A Queuing Model Approach to Staffing and Service Level Management

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ABSTRACT: ***In the realm of call centers, queuing models offer a valuable framework for analysis. In these models, customers are represented as callers, while servers take the form of call agents. Achieving an effective balance between service level and service costs is paramount for a call center's success, given the critical importance of service quality. This necessitates ensuring an adequate number of skilled agents at all times, a challenge commonly referred to as the staffing problem. This paper aims to elucidate the application of the queuing model in evaluating the performance of the multi-skilled call center and determining the optimal number of agents in each group. Also, an algorithm is adopted to solve the staffing model. Through a numerical example, we compute the steady-state probabilities, the service levels, the optimal number of agents in each group, and the minimum cost associated with the service level requirements to show the influential factors within the system.***

KEYWORDS: Queuing model, Multi-skill call center, Service level, Staffing problem.

Date of Submission: 16-2-2024 Date of acceptance: 05-07-2024

I. INTRODUCTION

A call center functions as a pivotal service network where customers await assistance from ser- vice providers, or agents. In today's business landscape, call centers play an increasingly crucial role, aiming to enhance customer satisfaction and establish a reputable brand image. These centers primarily offer customer service and technical support, striving to optimize service quality while minimizing costs and wait times. With advancements in technology, call centers have evolved to incorporate various channels, adding complexity to agent tasks as they must possess diverse skills to handle different customer needs efficiently. Gans et al. provided an illustrative case study featuring an international call center renowned for its proficiency in handling calls in multiple languages. The study examined scenarios in multi-skill call centers, particularly those with a restricted number of skill groups, typically three groups or fewer. Additionally, the research offered comprehensive insights into different design strategies, including V-design, M-design, W-design, and N-design (9). Ormeci presented the dynamic admission control problem in a call center with shared and dedicated service facilities (18). Bhulai et al.; Cezik & L'Ecuyer investigated scenarios involving a greater number of call types and skill groups within multi-skill call centers (4,5). Several studies have been conducted on telephone call centers, including surveys by Aksin et al.; Koole & Mandelbaum (1,11).

The use of queueing models is becoming increasingly important in call centers, where customers are callers and agents are servers. The challenge for call centers is to balance service levels with service costs while ensuring the quality of service remains high. This involves ensuring an appropriate number of multi-skilled agents are available to meet the expected demand, which is known as the staffing problem. Several studies have been conducted on staffing problems, including surveys by Aksin et al.; Dam et al.; Defraeye & Van Nieuwenhuyse; Koole & Pot (1,7,8,12). The staffing problem is discussed on M-design multi-skill call center with impatient customers by Li et al.; Priya & Rajendran (13,19). Li &Yue conducted research on multi-skill call centers employing the N-design approach, which entails categorizing agents into various groups according to their skills and employing skill-based routing to match customers with suitable agents (14). Other researches about this problem see e.g. Chevalier; Lia et al. (6,16). Several researchers have developed heuristic methods to optimize staffing in call centers, achieving efficient solutions with reduced computation time compared to traditional approaches, see Atlason et al.; Avramidis et al.; Horng & Lin; Li et al.; Lu et al.; Pot et al. (2,3,10,15,17,21).

This paper's structure is organized as follows: Section 2 described the multi-skill call center system model. Section 3 presented the division of the system's state space, the calculation of the state-transition rates based on the results of $M/M/c/c$ and $M/M/c$ queueing models, the calculation of the steady-state probabilities of the system through the establishment of the equilibrium equations, and the calculation of the call center's service level. Section 4 introduced the computational technique of the staffing problem. Section 5 explains the previous results through an example. Finally, concluding comments are given in Section 6.

II. MODEL DESCRIPTION

In our model, the calls (customers) are classified into two types (Call Type 1 and Call Type 2) and agents (servers) are divided into three groups (Group 1, Group 2, and Group 3) with different skills. This queueing model is fully characterized by customer profiles (Arrival Process), agent properties (Service Process), routing policies, and the limitation of the waiting queue (Queueing Discipline). The model is shown in Figure 1. Our methodology as follows:

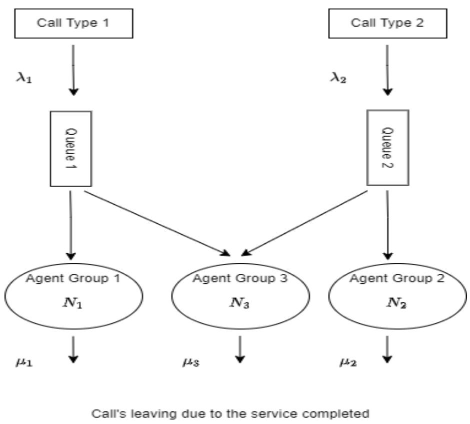

Figure 1. The multi-skill call center queue model

2.1. Customer profiles:

Two customer types arrive according to the Poisson process with arrival rates λ_1 and λ_2, respectively. There are two queues (Queue 1 and Queue 2), the arriving calls are lined with it, which contain calls of Type 1 and Type 2, respectively. There are infinite waiting spaces for two queues.

2.2. Agent properties:

There are three categories of agents. Group 1 (has skill 1), Group 2 (has skill 2) and Group 3 (have skill 1 and skill 2) consist of N_1 agents, N_2 agents and N_3 agents, respectively. Group 1 (who can only serve calls of Type 1) is of specialized agents with N_1 agents and with mean μ_1. Group 2 (who can only serve calls of Type 2) is also specialized agents with N_2 agents and with mean μ_2. Group 3 (who can serve calls of both Type 1 and Type 2) is of flexible agents with N_3 agents and with mean μ_3. The service times of agents in Group 1, Group 2, and Group 3 are all exponentially distributed.

2.3. Limitation of the waiting queue:

The waiting spaces for both two queues are infinite. Each type of calls has its own queue. The queues of both two call types (Call Type 1 and Call Type 2) are independent of each other. For the same type of waiting calls (Call Type 1 or Call Type 2), they are served in FCFS (First-come First- served) discipline by a free agent of its own group (Group 1 or Group 2) and also, a free agent in Group 3 serve the waiting calls in Queue 1 or Queue 2 according to FCFS discipline. If all agents in both Group 1 and Group 2 are busy and there are waiting for calls both in them, a free agent in Group 3 will pick out a Call Type 1 and Call Type 2 for service at random (i.e., with equal probability).

2.4. Routing Policy

The routing policy in our model is skill-based routing (i.e., it is based on skills). There are priorities for various call types. The waiting calls (customers) of Type 1 have priority to be served by the agent in Group 1 if there are free agents in Group 1 and free agents in Group 3. Also, the waiting calls of Type 2 have priority to be served by the agent in Group 2 if there are free agents in Group 2 and free agents in Group 3. The calls will be serviced by a free agent in Group 3 if all agents in Group 1 or Group 2 are busy. The customer must wait in Queue 1 or Queue 2 if all agents are busy in Group 3.

III. STEADY-STATE PROBABILITIES

In this section, we firstly defined the states of our system model, then obtained the state-transition rates in two cases (call arrival and service completion) and established the equilibrium equations for the steady-state probabilities of the model when the model is stationary. Finally, we computed the service level.

3.1. State space description:

We have two groups with various skills in our model wherein each of them there are three states (idle, busy, and overload). While the third group has two states (idle and busy).

I- An Idle State: In this case, at least one agent is idle. This state symbolized by 1.

II- A Busy State: In this case, all agents in the group are busy and there no calls waiting for service served by this group. This state symbolized by 2.

III- An Overload State: In this case, all agents in the group are busy, and there is at least one call waiting for service by this group. This state symbolized by 3.

We note that the state space of the system model consisted of 12 states, according to the routing policy assumed above. Thus, the state space is given by:

$$E = \{(1\ 1\ 1), (1\ 2\ 1), (1\ 2\ 2), (1\ 3\ 2), (2\ 1\ 1), (2\ 1\ 2), (2\ 2\ 2), (2\ 2\ 1), (2\ 3\ 2), (3\ 1\ 2), (3\ 2\ 2)\}$$

Let $S_i, (i = 1, 2, 3, \ldots, 12)$ be the i^{th} state in the state space E. let $n_j, (j = 1, 2)$ be the number of calls waiting in a queue that are assumed to be served by Group i plus the busy agents in Group i. While n_3 be the number of calls being serviced by agents of Group 3 as no waiting calls in a queue according to routing policy.

3.2. The construction of the state-transition rates:

We derive the state-transition rates by using the results of $M/M/c$ and $M/M/c/c$ queueing systems. There are only two events that can make the state transferred: call arrival or service fulfillment. We will be debating the two cases separately to obtain how the state transition rates are computed.

3.2.1. The transfer of states due to the call arrival:

We assume the state $S_1 = (111)$, for example, which means the agents in the three groups (Group 1, Group 2, and Group 3) are in the idle state. Let $q_{(i-j)}, (i, j = 1,2,3, \ldots ,12)$ denote the state-transition rate. The trigger for the transfer from the state S_1 to the state S_5 is due to Call Type 1, see Figure 2.

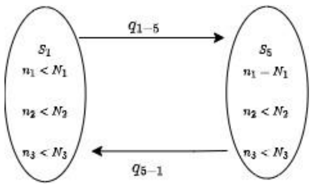

Figure 2. Diagram of the state-transition from the state S_1 to the state S_5

Thus, the transition rate from the state S_1 to the state S_5 is obtained as follows:

$$q_{1-5} = \lambda_1 P(n_1 = N_1 - 1) \quad (1)$$

Where $P(n_1 = N_1 - 1)$ is the probability that the number of Call Type 1 needing to be serviced by the agents in Group 1 is $n_1 = N_1 - 1$ for state S_1. Note that $n_1 < N_1$ and $n_2 < N_2$ (i.e., if the operation is in state S_1 then the number of calls of either Type 1 or Type 2 is less than the number of agents either in Group 1 or Group 2), and that the two queues are independent of each other so the results of the $M/M/c/c$ loss queueing system can be used. Consequently, we have

$$P(n_1 = N_1 - 1) = \frac{\rho_1^{N_1-1}}{(N_1-1)! \sum_{j=0}^{N_1} \frac{\rho_1{}^j}{j!}}; \rho_1 = \frac{\lambda_1}{\mu_1}, a_1 = \frac{\lambda_1}{N_1\mu_1} \quad (2)$$

In a similar way, we can acquire the other transition rates q_{i-j} caused by the arrival of calls as follows:

$$q_{1-5} = q_{2-8} = q_{3-7} = q_{4-9} = \lambda_1 P(n_1 = N_1 - 1) \quad (3)$$
$$q_{1-2} = q_{6-7} = q_{5-8} = q_{10-11} = \lambda_2 P(n_2 = N_2 - 1) \quad (4)$$
$$q_{9-12} = q_{7-11} = q_{6-10} = \lambda_1 \quad (5)$$
$$q_{7-9} = q_{3-4} = q_{11-12} = \lambda_2 \quad (6)$$
$$q_{5-6} = \lambda_1 P^2(n_3 = N_3 - 1) \quad (7)$$
$$q_{2-3} = \lambda_2 P^1(n_3 = N_3 - 1) \quad (8)$$
$$q_{8-11} = q_{8-9} = (\lambda_1 + \lambda_2) P^3(n_3 = N_3 - 1) \quad (9)$$
$$q_{1-8} = \lambda_1 P(n_1 = N_1 - 1) + \lambda_2 P(n_2 = N_2 - 1) \quad (10)$$
$$q_{7-12} = \lambda_1 + \lambda_2 \quad (11)$$

$$q_{3-9} = \lambda_1 P(n_1 = N_1 - 1) + \lambda_2 \quad (12)$$
$$q_{6-11} = \lambda_2 P(n_2 = N_2 - 1) + \lambda_1 \quad (13)$$
$$q_{2-7} = \lambda_1 P(n_1 = N_1 - 1) + \lambda_2 P^1(n_3 = N_3 - 1) \quad (14)$$
$$q_{5-7} = \lambda_2 P(n_2 = N_2 - 1) + \lambda_1 P^2(n_3 = N_3 - 1) \quad (15)$$

Where

$$P(n_2 = N_2 - 1) = \frac{\rho_2^{N_2-1}}{(N_2-1)!\sum_{j=0}^{N_2}\frac{\rho_2{}^j}{j!}}; \rho_2 = \frac{\lambda_2}{\mu_2}, a_2 = \frac{\lambda_2}{N_2\mu_2} \quad (16)$$

$$P^1(n_3 = N_3 - 1) = \frac{\rho_3^{N_3-1}}{(N_3-1)!\sum_{j=0}^{N_3}\frac{\rho_3{}^j}{j!}}; \rho_3 = \frac{\lambda_1}{\mu_3} \quad (17)$$

$$P^2(n_3 = N_3 - 1) = \frac{\rho_4^{N_3-1}}{(N_3-1)!\sum_{j=0}^{N_3}\frac{\rho_4{}^j}{j!}}; \rho_4 = \frac{\lambda_2}{\mu_3} \quad (18)$$

$$P^3(n_3 = N_3 - 1) = \frac{(\rho_3+\rho_4)^{N_3-1}}{(N_3-1)!\sum_{j=0}^{N_3}\frac{(\rho_3+\rho_4)^j}{j!}} \quad (19)$$

3.2.2. The transfer of states due to the fulfillment of service:

We consider the state $S_2 = (121)$, for example. If the call of Type 2 finished the service, then the set of states will be varied from state S_2 to state S_1. In the state S_2, $n_2 = N_2$ (i.e., all N_2 agents are busy) and the service rate for the call of Type 2 is $N_2\mu_2$. The trigger for the transfer from state S_2 to state S_1 is obtained as follows (see Figure 2):

$$q_{2-1} = N_2\mu_2 \quad (20)$$

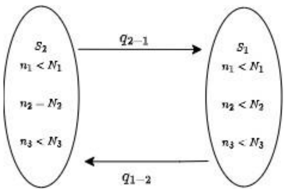

Figure 3. Diagram of the state-transition from the state S_2 to the state S_1

In a similar way, we can acquire the other transition rates q_{i-j} caused by the arrival of calls as follows:

$$q_{5-1} = q_{8-2} = q_{7-3} = q_{9-4} = N_1\mu_1 \quad (21)$$
$$q_{2-1} = q_{7-6} = q_{8-5} = q_{11-10} = N_2\mu_2 \quad (22)$$
$$q_{3-2} = q_{6-5} = N_3\mu_3 \quad (23)$$
$$q_{12-9} = q_{11-7} = q_{10-6} = N_1\mu_1 P(n_1 = N_1 + 1) \quad (24)$$
$$q_{9-7} = q_{4-3} = q_{12-11} = N_2\mu_2 P(n_2 = N_2 + 1) \quad (25)$$
$$q_{9-3} = N_1\mu_1 + N_2\mu_2 P(n_2 = N_2 + 1) \quad (26)$$
$$q_{11-6} = N_2\mu_2 + N_1\mu_1 P(n_1 = N_1 + 1) \quad (27)$$
$$q_{7-2} = N_1\mu_1 + N_3\mu_3 \quad (28)$$
$$q_{8-1} = N_2\mu_2 + N_3\mu_3 \quad (29)$$
$$q_{12-7} = N_1\mu_1 P(n_1 = N_1 + 1) + N_2\mu_2 P(n_2 = N_2 + 1) \quad (30)$$
$$q_{11-8} = (N_1\mu_1 + \frac{1}{2}N_3\mu_3)P(n_1 = N_1 + 1) \quad (31)$$
$$q_{9-8} = (N_2\mu_2 + \frac{1}{2}N_3\mu_3)P(n_2 = N_2 + 1) \quad (32)$$

The results of the $M/M/c$ queueing system can be used to obtain the probabilities of $P(n_1 = N_1 + 1)$ and $P(n_2 = N_2 + 1)$ as follows:

$$P(n_1 = N_1 + 1) = \frac{\rho_1^{N_1+1}}{N_1(N_1)!}P_0^1, P_0^1 = [\sum_{j=0}^{N_1-1}\frac{\rho_1{}^j}{j!} + \frac{N_1\rho_1{}^{N_1}}{N_1!(N_1-\rho_1)}]^{-1};$$
(33)

$$P(n_2 = N_2 + 1) = \frac{\rho_2^{N_2+1}}{N_2(N_2)!}P_0^2, P_0^2 = [\sum_{j=0}^{N_2-1}\frac{\rho_2{}^j}{j!} + \frac{N_2\rho_2{}^{N_2}}{N_2!(N_2-\rho_2)}]^{-1};$$
(34)

3.3. The computation of steady-state probabilities:

The steady-state probabilities of each state are denoted by $P_i, (i = 1,2,3, \ldots ,12)$ and the equations of the steady-state probabilities of the system are shown as follows:

$$P_1(q_{1-2} + q_{1-5} + q_{1-8}) = P_2 q_{2-1} + P_5 q_{5-1} + P_8 q_{8-1} \quad (35)$$
$$P_2(q_{2-1} + q_{2-3} + q_{2-7} + q_{2-8}) = P_1 q_{1-2} + P_3 q_{3-2} + P_7 q_{7-2} + P_8 q_{8-2} \quad (36)$$
$$P_3(q_{3-4} + q_{3-7} + q_{3-9}) = P_4 q_{4-3} + P_7 q_{7-3} + P_9 q_{9-3} \quad (37)$$

$$P_4(q_{4-3}+q_{4-9}) = P_3q_{3-4} + P_9q_{9-4} \quad (38)$$
$$P_5(q_{5-1}+q_{5-6}+q_{5-7}+q_{5-8}) = P_1q_{1-5} + P_6q_{6-5} + P_7q_{7-5} + P_8q_{8-5} \quad (39)$$
$$P_6(q_{6-5}+q_{6-7}+q_{6-10}+q_{6-11}) = P_5q_{5-6} + P_7q_{7-6} + P_{10}q_{10-6} + P_{11}q_{11-6} \quad (40)$$
$$P_7(q_{7-2}+q_{7-3}+q_{7-5}+q_{7-6}+q_{7-9}+q_{7-11}+q_{7-12}) = P_2q_{2-7} + P_3q_{3-7} + P_5q_{5-7} + P_6q_{6-7} + P_9q_{9-7} + P_{11}q_{11-7} + P_{12}q_{12-7} \quad (41)$$
$$P_8(q_{8-1}+q_{8-2}+q_{8-5}+q_{8-9}+q_{8-11}) = P_1q_{1-8} + P_2q_{2-8} + P_5q_{5-8} + P_9q_{9-8} + P_{11}q_{11-8} \quad (42)$$
$$P_9(q_{9-3}+q_{9-4}+q_{9-7}+q_{9-8}+q_{9-12}) = P_3q_{3-9} + P_4q_{4-9} + P_7q_{7-9} + P_8q_{8-9} + P_{12}q_{12-9} \quad (43)$$
$$P_{10}(q_{10-6}+q_{10-11}) = P_6q_{6-10} + P_{11}q_{11-10} \quad (44)$$
$$P_{11}(q_{11-6}+q_{11-7}+q_{11-8}+q_{11-10}+q_{11-12}) = P_6q_{6-11} + P_7q_{7-11} + P_8q_{8-11} + P_{10}q_{10-11} + P_{12}q_{12-11} \quad (45)$$
$$P_{12}(q_{12-7}+q_{12-9}+q_{12-11}) = P_7q_{7-12} + P_9q_{9-12} + P_{11}q_{11-12} \quad (46)$$
$$\sum_{i=1}^{12} P_i = 1 \quad (47)$$

Since the above equations are linear, so by solving these equations, (using MATLAB software), we can obtain all the steady-state probabilities.

3.4. The computation of service level:

Calculating service level is used in our paper to evaluate the performance of a multi-skill call center and is calculated by using steady-state probabilities. The service level can be expressed as the percentage of the calls that should be serviced within a given waiting time T_i, denoted as $\frac{P_i}{T_i}$.

Let $P_{sl}^i = 1 - P_{ns}^i$, (i = 1, 2, 3) be the probability that the call of Type i is serviced in a fixed time T_i. Consider the call of Type 1, for example. Calls of Type 1 have a queue only occur in the states $S_{10} = (3\ 1\ 2)$, $S_{11} = (3\ 2\ 2)$, and $S_{12} = (3\ 3\ 2)$. The service rate for the call of Type 1 in each state of S_{10} and S_{11} is $N_1\mu_1 + N_3\mu_3$, the number of calls could be served in time T_1 is $T_1(N_1\mu_1 + N_3\mu_3)$. Also, the service rate in state S_{12} is $(N_1\mu_1 + \frac{1}{2}N_3\mu_3)$ according to the routing policy and the number of calls could be served in this state in time T_1 is $T_1(N_1\mu_1 + \frac{1}{2}N_3\mu_3)$. We can obtain the probability P_{ns}^1 via the above analysis as follows:

$$P_{ns}^1 = (P_{10} + P_{11})\sum_{i=K_1}^{\infty} P(n_1 = i) + P_{12}\sum_{i=K_2}^{\infty} P(n_1 = i) \quad (48)$$

Where

$$K_1 = N_1 + N_3 + T_1(N_1\mu_1 + N_3\mu_3),\ K_2 = N_1 + \frac{1}{2}N_3 + T_1(N_1\mu_1 + \frac{1}{2}N_3\mu_3) \quad (49)$$

Similarly, for a call of Type 2, calls have a queue only occur in the sates $S_4 = (1\ 3\ 2)$, $S_9 = (2\ 3\ 2)$, and $S_{12} = (3\ 3\ 2)$. Thus, we can obtain the probability P_{ns}^2 via the same analysis method above as follows:

$$P_{ns}^2 = (P_4 + P_9)\sum_{i=K_3}^{\infty} P(n_2 = i) + P_{12}\sum_{i=K_4}^{\infty} P(n_2 = i) \quad (50)$$

Where

$$K_3 = N_2 + N_3 + T_2(N_2\mu_2 + N_3\mu_3),\ K_4 = N_2 + \frac{1}{2}N_3 + T_2(N_2\mu_2 + \frac{1}{2}N_3\mu_3) \quad (51)$$

And also, $P(n_1 = i)$, and $P(n_2 = i)$ are the probabilities that there are i customers in the $M/M/N_1$, and $M/M/N_2$ queueing system with arrival rate λ_1, and λ_2 and service rates μ_1, and μ_2, respectively, which their formulas are given in (20). By using MATLAB software, we can get the service levels P_{sl}^1, and P_{sl}^2.

IV. STAFFING OPTIMIZATION

In this section, we offered the optimization model of the staffing problem to find the optimal numbers of the agents in each group to minimize the cost of the system. We suppose that the cost of the agents' Group 1 is C_1, the cost of the agents' Group 2 is C_2, and the cost of the agents' Group 3 is C_3. We seek to get the optimal number of agents N_1, N_2, and N_3 subject to the conditions of the constraint to minimize the cost of the system model. The staffing optimization can be expressed as follows:

$$\begin{aligned} \min z = {} & C_1N_1 + C_2N_2 + C_3N_3 \\ s.t.,\quad & P_{sl}^1 \geq \alpha_1, \\ & P_{sl}^2 \geq \alpha_2, \\ & a \leq N_i \leq b,\ a, b, N_i \epsilon Z^+; i = 1,2,3. \end{aligned} \quad (52)$$

where α_1, and α_2 are the given service rate of the Call Type 1, and the Call Type 2 respectively, Z^+denote the set of positive integers. The number of agents within each group may be chosen within the interval of (a, b). This problem is nonlinear integer programming with a linear objective function. The optimization involves determining four variables, namely, N_1, N_2, and N_3. The constraints pertain to incoming calls aimed at meeting specific service level criteria. Notably, the constraints are intricately nonlinear, as evident from the service level formula. Given the involvement of three variables in this model, an effective resolution can be achieved through the application of the specific algorithm using MATLAB software to obtain the optimal numbers of every agent group and the minimum cost associated with it. The specific algorithm is described as follows:

EXAMPLE

Algorithm 1 Staff Problem Algorithm

Inputs: $C_1, C_2, C_3, a, b, \alpha_1, \alpha_2, \lambda_1, \lambda_2, \mu_1, \mu_2, \mu_3, T_1, T_2$.
Outputs: The optimal number of agents in each group and minimum cost.
Step 1: Initialize the parameters.
Step 2: Select all variable combinations of N_i's such that $a \leq N_i \leq b$; $i = 1, 2, 3$.
Step 3: Compute all transition rates (from eqn. (1) to (34)), the steady-state probabilities (from eqn. (35) to (47)), and then the service levels (from eqn. (48) and (50)).
Step 4: Compare P^i_{sl} and α_i; $i = 1, 2$.
(a) If $P^i_{sl} \geq \alpha_i$, go to step (5).
(b) If $P^i_{sl} < \alpha_i$ for any i, go to step (2).
Step 5: Determine the number of agents in each group and compute the cost from the eqn.
$\min Z = C_1N_1 + C_2N_2 + C_3N_3$.
Step 6: Repeat step (2) until all selections of variable combinations of N_i's occur.
Step 7: Compare all previous cost in step (5) and then find the minimum cost, $N_1, N_2, and N_3$.
EXIT.

We present this numerical example to determine how these factors, such as the service levels and the number of agents in each group, influence the system model. To show how the results in Tables (1, 2, 3) were obtained, let us take case (1), for example, when $N_1 = 11$, $N_2 = 16$, and $N_3 = 18$. By giving the MATLAB software and applying the specific algorithm mentioned in the previous section, the parameter settings are as follows: $\lambda_1 = 5$, $\lambda_2 = 4$, $\mu_1 = 0.8$, $\mu_2 = 0.6$, $\mu_3 = 0.4$, $T_1 = 20$, $T_2 = 30$, $\alpha_1 = 0.8$, $\alpha_2 = 0.7$, $C_1 = 10$, $C_2 = 20$, $C_3 = 30$, $(a, b) = (11, 100)$, and utilizing the equations from (1) to (50) we get all the state-transition rates, then by substituting in equations from (2) to (47), we get all the steady-state probabilities, and by using equations from (48) to (51), we calculate the service levels, and from equation (52) we get the optimal number of servers in each group. By the same manner, we can get the other cases. Based on the results in Table (1), we can see that all steady-state probabilities are present in small-sized cases (50 or less agents), while in medium (51-200 agents) or large (more than 201 agents)-sized call centers, some steady-state probabilities are absent in each case, indicating a reduction in the complexity of the system as it scales. Comparing probabilities across cases reveals patterns and variations in system behavior, providing insights into resource utilization and performance. The table (2) demonstrates that the service level formula is applicable to call centers of any scale. In multi-skill call centers, the total number of agents remains constant, but the composition of agent groups significantly impacts the overall service level. Hence, optimizing the numbers of each agent group is crucial to achieve the desired service level while minimizing system costs. As shown in Table (3), the table shows that the optimal number of agents in each group varies for each case, and it also affects the system cost. It demonstrates that the system cost can be minimized by adjusting the number of agents in each group based on the service rate and cost for each group.

Table 1. Numerical results of the steady-state probabilities

Case	1	2	3	4	5	6

Case	N	N_1	N_2	N_3	P^1_{sl}	P^2_{sl}
1	45	11	16	18	0.8184	0.8493
2	45	11	15	19	0.8406	0.8576
3	90	30	30	30	0.9875	0.9848
4	90	40	20	30	0.9845	0.9904
5	210	100	70	40	1.0000	0.9998
6	210	90	80	40	0.9998	1.0000

Table 2. Numerical results of the service levels

Case	N_1	N_2	N_3	Cost
1	11	16	18	970
2	11	15	19	980
3	30	30	30	1800
4	40	20	30	1900
5	100	70	40	3600
6	90	80	40	3800

V. CONCLUSIONS

In this paper, we discussed the multi-skill call center based on queueing model (the exponential model in a multi-skill call center). The model system has two customer classes and three groups of agents where specialized agents can treat only their own customer type while flexible agents manage all types. We showed the state space of the system and by using the results of the $M/M/c/c$ and $M/M/c$ queueing system, we obtained the transition rates of the state sets, and then we created the equilibrium equations for the steady-state equations of the system. Also, we offered the computational formula of the service level, debated the computational technique of the staffing problem to calculate the optimal number of agents in each group.

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