Bulletin of Faculty of Science ,Zagazig University (BFSZU) 2024
Bulletin of Faculty of Science, Zagazig University (BFSZU)
e-ISSN: 1110-1555
Volume-2024, Issue-3, pp-131-139
https://bfszu.journals.ekb.eg/journal
Research Paper DOI: 10.21608/bfszu.2024.260691.1357

===

IRC Controller for Suppressing the Vibrations of Cantilever Beam Excited by an External Force

Y. A. Amer[1], A. T. EL-Sayed[2], M. N. Abd EL-Salam[3] and S. S. Mahmoud[1*]

1 Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt
2 Department of Basic Sciences, Modern Academy for Engineering and Technology, Maadi, Egypt
3 Department of Basic Sciences, Higher Technological Institute, 10th of Ramadan City, Egypt

ABSTRACT **: Vibration of structures is often an unwanted phenomena and should be avoided or controlled. In this paper, we study the effect of an integral resonant controller (IRC) on reducing the vibrations of the cantilever beam excited by an external force. The suggested system has two degree of freedom which is a non-linear system with the fifth and cubic nonlinearity terms excited by an external force. The second order approximate solutions of the system equation are sought using the multiple scale perturbation (MSPT). The frequency response equation is studied to test the behavior of the steady state solutions at the primary resonance case ($\Omega \cong \omega$). The behavior of uncontrolled and controlled system is presented using time histories. Also, the stability of the system is investigated applying frequency response equation using the Runge-Kutta fourth order method. To scrutinize the time histories of the system before and after using IRC. The effects of different parameters of the system are studied numerically.**

KEYWORDS: Cantilever beam, Multiple scale and Integral resonant controller.

Date of Submission: 09-01-2024 Date of acceptance: 14-03-2024

I. INTRODUCTION

A many studied showed that the resonance of the nonlinear system and its effect on the stability. Nayfeh and Mook [10] studied the nonlinear systems with primary and secondary resonance. Boru [5] explained the stability of Laval rotor with noncircular shaft. In the other work of Ref. [6] used the internal resonance to suppress the vibration of the cantilever beam excited by under chord wise and decreased blending of the response curve is due to the nonlinearity. Sadri and Younesian [12] demonstrated the nonlinear harmonic vibration acting on a plate cavity system in cases: primary, secondary and combination of resonances and discussed their influence on the stability of the system.

Nonlinear oscillations can be reduced [8, 14], using the time delayed to suppress the vibration of the structure, such as linear and nonlinear time delayed position feedback control motions of a van-der pol. In Ref. [9] abled to control in the vibrations of the cantilever beam by time delay controller state feedback and discussed the stability in case primary resonance. In [15], the authors introduced the velocity feedback control to suppress the vibration of the cantilever beam. Saeed et al. [13] investigated suppression of the nonlinear dynamic system by the time delay saturation based controller and analyzed the approximate solution and numerical solution. Omidi and Mahmoodi [11] described that the effect of nonlinear integral positive position feedback (NIPPF) and the integral resonant controller (IRC) and positive position feedback (PPF) on the cantilever beam also, they showed that the effect of (NIPPF) was the best of controller to reduce the response curve. In [8], Li et al. used the time delay and velocity feedback as the active control of the cantilever beam in two cases: primary and secondary resonance with an intermediate lumped mass. Kandi and EL-Gohary [7], presented a method to reduce the oscillation resulted to rotating speed using the nonlinear saturation controller. Atkinson et al. [1] demonstrated the numerical solution of ordinary differential equations a lot of ways, including Runge-kutta methods and discussed the stability of those methods.

Recently, Bauomy et al. [4] made a combination of IRC and nonlinear saturation controller (NSC) to improve the vibrational behaviors of a cantilever beam model through an intermediate lumped mass. Amer et al. [2] achieved results that the effect the nonlinear positive position feedback on a hybrid Rayleigh-van –der –pol doffing oscillator. In [3], the authors investigated four type of controller on the vibration of a cantilever beam that the negative linear velocity feedback, the negative cubic velocity feedback, NSC and positive position feedback.

In this paper, we study the effect of integral resonant controller on the vibration of a cantilever beam with cubic and fifth nonlinearity terms excited by an external force by using the MSPT. The numerical results show that after using the control the resonance caused by nonlinear borders is reduced. Effects of some different behavior parameters and numerical comparison results is illustrated

.II. MATERIALS AND METHODS

The equation of motion of a cantilever beam with (IRC) described by the following differential equation

$$\ddot{x}+\hat{\alpha}\dot{x}+\hat{\beta}_1\dot{x}^3+\hat{\beta}_2\dot{x}^5+\omega^2 x+\hat{\gamma}_1 x^3+\hat{\gamma}_2 x^5+\hat{\delta}_1(x\dot{x}^2+x^2\ddot{x})+\hat{\delta}_2(x^3\dot{x}^2+x^4\ddot{x})=\hat{f}\cos\Omega t+\hat{J}z, \quad (1)$$

$$\dot{z}+\eta z=\gamma x, \quad (2)$$

where x is the displacement of the cantilever beam , ω is the natural frequency, Ω is the excitation frequency , η and γ are the integrator gain.

To execute the solution procedure, the system parameters should be scaled as follows:

$$\hat{\alpha}=\varepsilon\alpha, \hat{\beta}_i=\varepsilon\beta_i, \hat{\gamma}_i=\varepsilon\gamma_i, \hat{\delta}_i=\varepsilon\delta_i, \hat{f}=\varepsilon f, \hat{J}=\varepsilon J, i=1,2$$

where ε is a very small perturbation parameter(i.e. $\varepsilon \ll 1$)

$$\ddot{x}+\varepsilon\alpha\dot{x}+\varepsilon\beta_1\dot{x}^3+\varepsilon\beta_2\dot{x}^5+\varepsilon\omega^2 x+\varepsilon\gamma_1 x^3+\varepsilon\gamma_2 x^5+\varepsilon\delta_1(x\dot{x}^2+x^2\ddot{x})+\varepsilon\delta_2(x^3\dot{x}^2+x^4\ddot{x})$$
$$=\varepsilon f\cos\Omega t+\varepsilon Jz, \quad (3)$$

where, α represents the damping coefficient, β_i ,γ_i and $\delta_i(\mathrm{i}=1,2)$ are nonlinearities terms coefficients. The parameter f is the amplitude of the excitation force.

Perturbation techniques

We seek a uniform approximate solution to (2) of the form

$$x(t,\varepsilon)=x_0(T_0,T_1)+\varepsilon x_1(T_0,T_1)+O(\varepsilon^2), \text{ and} \quad (4)$$

$$z(t,\varepsilon)=z_o(T_o,T_1)+\varepsilon z_1(T_o,T_1). \quad (5)$$

We note that the $T_n(n=0,1\ldots)$ represent different time scale are defined by $T_0=t$ and $T_1=\varepsilon t\ldots$

$$\frac{d}{dt}=D_0+\varepsilon D_1+\ldots \quad \frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0 D_1+\ldots \quad (6)$$

Substituting from (4) and (5) into (2) and (3) equating each of the coefficients of $\varepsilon^0$and ε to zero, we get

$$(D_0^2+\omega^2)x_0=0 \quad (7)$$

$$(D_0+\eta)z_0=\gamma x_0 \quad (8)$$

$$(D_0^2+\omega^2)x_1=-2D_0D_1x_0-\alpha D_0x_0-\beta_1(D_0x_0)^3-\beta_2(D_0x_0)^5$$
$$-\gamma_1x_0^3-\gamma_2x_0^5-\delta_1[x_0(D_0x_0)^2+x_0(D_0^2x_0)]-\delta_2[x_0^3(D_0x_0)^2$$
$$+x_0^4(D_0^2x_0)]+f\cos\Omega T_0+Jz_0. \quad (9)$$

The general solution of Eq. (7) is

$$x_0=\mathrm{A}\exp(i\omega T_o)+\mathrm{C.C} \quad (10)$$

We use Eq. (10) to obtain the solution of equation (8)

$$z_o=\mathrm{B}e^{-\eta T_0}+\frac{\gamma\mathrm{A}(\eta-i\omega)}{\eta^2+\omega^2}\exp(i\omega T_0)+\mathrm{C.C}, \quad (11)$$

where A and B are complex functions in$\mathrm{T}_1$and C.C denotes the complex conjugate terms.

Substituting from Eq. (10) and (11) into (9), we get

$$(D_0{}^2+\omega^2)x_1=(-2i\omega D_1\mathrm{A}\text{-}i\alpha\omega\mathrm{A}-3i\beta_1\omega^3\mathrm{A}^2\bar{\mathrm{A}}\text{-}10i\beta_2\omega^5\mathrm{A}^3\bar{\mathrm{A}}^2-3\gamma_1\mathrm{A}^2\bar{\mathrm{A}}$$
$$\text{-}10\gamma_2\mathrm{A}^3\bar{\mathrm{A}}^2+2\delta_1\omega^2\mathrm{A}^2\bar{\mathrm{A}}+8\delta_2\omega^2\mathrm{A}^3\bar{\mathrm{A}}^2+\frac{\gamma\mathrm{A}(\eta-i\omega)}{\eta^2+\omega^2})\exp(i\omega T_0)$$
$$+(i\beta_2\omega^5\mathrm{A}^5-\gamma_2\mathrm{A}^5+2\delta_2\omega^2\mathrm{A}^5)\exp(5i\omega T_0)+((5i\beta_1\omega^5-5\gamma_2+6\delta_2\omega^2)\mathrm{A}^4\bar{\mathrm{A}}$$
$$+(i\beta_1\omega^3-\gamma_1+2\delta_1\omega^2)\mathrm{A}^3)\exp(3i\omega T_0)+\frac{f}{2}\exp(i\Omega T_0)+\mathrm{B}\mathrm{J}\exp(-\eta T_0)+\mathrm{C.C} \qquad (12)$$

The particular solution of Eq. (1.12) is

$$x_1=\mathrm{E}_1\exp(3i\omega T_0)+\mathrm{E}_2\exp(5i\omega T_0)+\frac{f}{2}\exp(i\Omega T_0)+\mathrm{B}J\exp(-\eta T_0)+\mathrm{C.C} \qquad (13)$$

where $\mathrm{E}_1$and E_2 are complex functions inT_1.

From the solution derived above, several resonance case can be extracted. Resonance reported in this approximation order is primary resonance: $\Omega\cong\omega$.

Stability of steady state solution

For stability of system is investigated at primary resonance $\Omega\cong\omega$.by eliminating the terms of Eq. (12) not to produce the secular terms in Eq. (13), and convert small-divisors into secular terms by $\Omega=\omega+\varepsilon\sigma$, one finds the solvability conditions

$$-2i\omega D_1\mathrm{A}-i\alpha\omega\mathrm{A}-3i\beta_1\omega^3\mathrm{A}^2\bar{\mathrm{A}}-10\beta_2\omega^5\mathrm{A}^3\bar{\mathrm{A}}^2+(2\delta_1\omega^2-3\gamma_1)\mathrm{A}^2\bar{\mathrm{A}}+(8\delta_2\omega^3-10\gamma_2)\mathrm{A}^3\bar{\mathrm{A}}^2$$
$$+\mathrm{J}\gamma\mathrm{A}\frac{(\eta-i\omega)}{(\eta^2+\omega^2)}+\frac{f}{2}\exp(i\sigma\mathrm{T}_1)=0 \qquad (14)$$

The polar form of function A can be expressed as

$$\mathrm{A}=\frac{1}{2}a(T_1)\exp(i\zeta) \qquad (15)$$

where a is the steady-state amplitude. Inserting Eq. (15) into Eq. (14) and separating real and imaginary parts. We obtain

$$\dot{a}=\frac{-1}{2}\alpha a-\frac{3}{8}\beta_1\omega^2a^3-\frac{5}{16}\beta_2\omega^4a^5-\frac{\gamma\mathrm{J}a}{2(\eta^2+\omega^2)}+\frac{f}{\omega}\sin(\varphi) \qquad (16)$$

$$\dot{\zeta}=\frac{-1}{8\omega}(2\delta_1\omega^2-3\gamma_1)a^2+\frac{-1}{16\omega}(4\delta_2\omega^2-5\gamma_2)a^4-\frac{\gamma\eta\mathrm{J}}{2(\eta^2+\omega^2)}-\frac{f}{2\omega a}\cos(\varphi) \qquad (17)$$

where $\varphi=\sigma\mathrm{T}_1-\zeta$,the Eq.(17)becomes:

$$\dot{\varphi}=\sigma+\frac{1}{8\omega}(2\delta_1\omega^2-3\gamma_1)a^2+\frac{1}{16\omega}(4\delta_2\omega^2-5\gamma_2)a^4+\frac{\gamma\eta\mathrm{J}}{2(\eta^2+\omega^2)}+\frac{f}{2\omega a}\cos(\varphi) \qquad (18)$$

Equilibrium solution

At steady-state motion we have:

$\dot{a}=\dot{\varphi}=0$

Substituting Eq. (17) and (18) we get:

$$\sin(\varphi)=\frac{\omega}{f}(\frac{1}{2}\alpha a+\frac{3}{8}\beta_1\omega^2a^3+\frac{5}{16}\beta_2\omega^4a^5+\frac{\gamma\mathrm{J}a}{2(\eta^2+\omega^2)}) \qquad (19)$$

$$\cos(\varphi)=\frac{2\omega a}{f}(\frac{-1}{8\omega}(2\delta_1\omega^2-3\gamma_1)a^2+\frac{-1}{16\omega}(4\delta_2\omega^2-5\gamma_2)a^4-\frac{\gamma\eta\mathrm{J}}{2(\eta^2+\omega^2)}) \qquad (20)$$

Finally, to derive the steady-state frequency response, we are squaring Eqs. (19) and (20) then adding:

$$(\frac{1}{4}\alpha a+\frac{3}{8}\beta_1\omega^2a^3+\frac{5}{32}\beta_2\omega^4a^5+\frac{\gamma \mathrm{J}a}{4(\eta^2+\omega^2)})^2+(\frac{-1}{8\omega}(2\delta_1\omega^2-3\gamma_1)a^2+\frac{-1}{16\omega}(4\delta_2\omega^2-5\gamma_2)a^4$$
$$-\frac{\gamma\eta \mathrm{J}}{2(\eta^2+\omega^2)})^2=\frac{f^2}{4\omega^2} \quad (21)$$

The stability of a particular equilibrium solution was determined of jacobian matrix of the right-hand side of Eq. (12). If the real part of each eigenvalue is negative, the corresponding equilibrium solution is asymptotically stable. If the real part is positive, the corresponding equilibrium solution is unstable.to evaluate the stability we assume that

$$a=a_0+a_1,\varphi=\varphi_0+\varphi_1 \quad (22)$$

where $a_0$and φ_0 satisfies Eq.(19)and (20), $a_1$andφ_1 are perturbations which are assumed to be small compared to $a_0$andφ_0. Substituting Eq. (21) into Eqs. (16) and (18) we keep only the linear terms of $a_1$andφ_1, we have

$$\dot{a}_1=\mathrm{r}_{11}a_1+\mathrm{r}_{12}\varphi_1 \quad (23)$$

$$\dot{\varphi}_1=\mathrm{r}_{21}a_1+\mathrm{r}_{22}\varphi_1 \quad (24)$$

where r_{ij}(i=1,2)and(j=1,2)are given in the appendix. Eqs. (22)and(23)
can be represented in the matrix form as:

$$[\dot{a}_1,\dot{\varphi}_1]^{\mathrm{T}}=[\mathrm{R}][a_1,\varphi_1]^{\mathrm{T}} \quad (25)$$

Where

$$\mathrm{r}_{11}=-(\frac{\alpha}{2}+\frac{9}{8}\beta_1\omega^2a^2+(\frac{25}{16})\beta_2\omega^4a^4+\frac{\mathrm{J}\gamma}{2(\eta^2+\omega^2)})$$

$$\mathrm{r}_{12}=\frac{f}{2\omega}\cos\varphi$$

$$\mathrm{r}_{21}=-\frac{3}{4\omega}a\gamma_1-\frac{5}{4\omega}a^3\gamma_2+\frac{\delta_1}{2}a\omega+a^3\delta_2\omega-\frac{f}{2\omega a^2}\cos\varphi$$

$$\mathrm{r}_{22}=-\frac{f}{2\omega a}\sin\varphi$$

$$[\mathrm{R}]=\begin{bmatrix}\mathrm{r}_{11} & \mathrm{r}_{12}\\ \mathrm{r}_{21} & \mathrm{r}_{22}\end{bmatrix}$$

is the jacobian matrix.

Thus the stability of the steady-state motion depends on the eigenvalues of the jacobian matrix. We can obtain the following eigenvalue equation

$$\begin{vmatrix}\mathrm{r}_{11}-\lambda & \mathrm{r}_{12}\\ \mathrm{r}_{21} & \mathrm{r}_{22}-\lambda\end{vmatrix}=0. \quad (26)$$

where
Expanding this determinant then the eigenvalues can be determined by the following stability polynomial:

$$\lambda^2-(\mathrm{r}_{11}+\mathrm{r}_{12})\lambda+\mathrm{r}_{11}\mathrm{r}_{22}-\mathrm{r}_{12}\mathrm{r}_{21}=0. \quad (27)$$

where λ denotes eigenvalues of matrix [R], For the above system's solution to be stable, the Routh-Hurwitz criterion must be satisfied such that:

$$\mathrm{r}_{11}+\mathrm{r}_{22}>0 \quad \text{and} \quad \mathrm{r}_{11}\mathrm{r}_{22}-\mathrm{r}_{12}\mathrm{r}_{21}>0 \quad (28)$$

III. RESULTS AND DISCUSSION

Numerical consequence

In this part, we use the Runge-kutta fourth order method to estimate the numerical solution. Figure1: shows the responses of the model without control process at$\Omega\cong\omega$ and zero initial:

$\omega = 10, \alpha = 0.16, \beta_1 = 0.3331, \beta_2 = 0.1299, \gamma_1 = 22.5, \gamma_2 = 0.1319,$
$\delta_1 = -4.5, \delta_2 = 2.2, f = 0.5, J = 10, \gamma = 10, \eta = 10.$

The influence of controller appears in reduction of the Jump-Phenomenon caused by the cubic and fifth nonlinearity terms, the open-loop curve denotes the response curve of the system without control (i.e. $J = 0, \gamma = 10, \eta = 0$) and the closed-loop curve denotes the effect the control on the system.

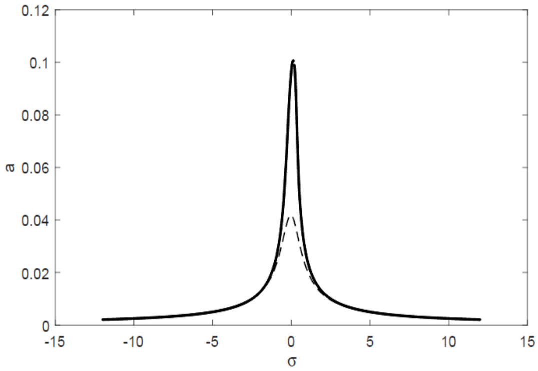

Figure1: Comparison between the system with control and without control.

Time history

We show the time history of the system without control at primary resonance in figure2.We can see from this figure that the system's response is approximately 0.1.the vibrations decrease by 0.04.thus, the effectiveness of the IRC is 60%.figure3, illustrates the results when the controller is effective, when $\Omega \cong \omega$.the effectiveness of the controller is E_a (E_a =steady state amplitude of the main system without controller / steady state amplitude of the main system with controller) is about 2.5.

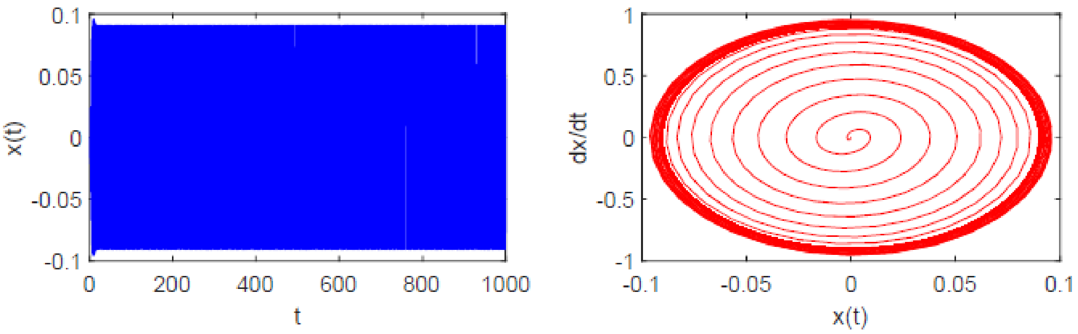

Figure2: time history of the main system at primary resonance.

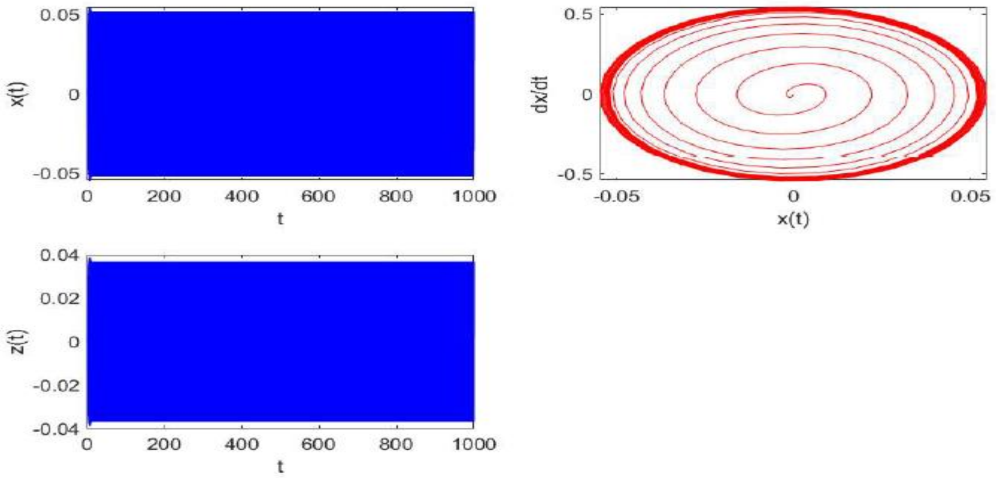

Figure3: The time history of the system with control.

Response curves

Eq. (21), solved numerically to obtain the graphical solution for the amplitudes of both cantilever beam and the IRC controller via the detuning parameterσ.As shown in figure4.

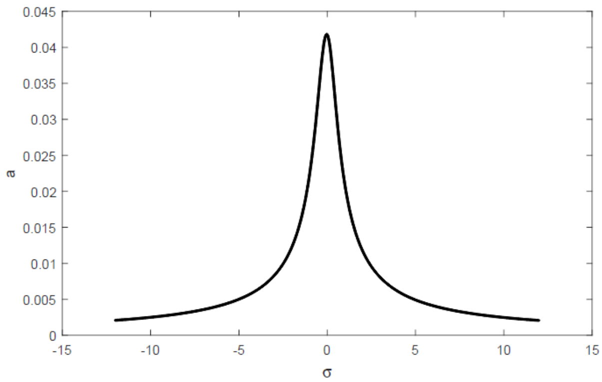

Figure4: the response curve of the system with control.

The response curve of the system with control decreases when natural frequency ω and damping coefficient α increases as illustrated in figure 5A and B. figure 5C and D shows that the amplitude of the response curve increases, when the increasing in the excited force *f* and the integrator gain η. When the nonlinear parameterβ_1 increases, the response curve decreases as shown in figure5E.

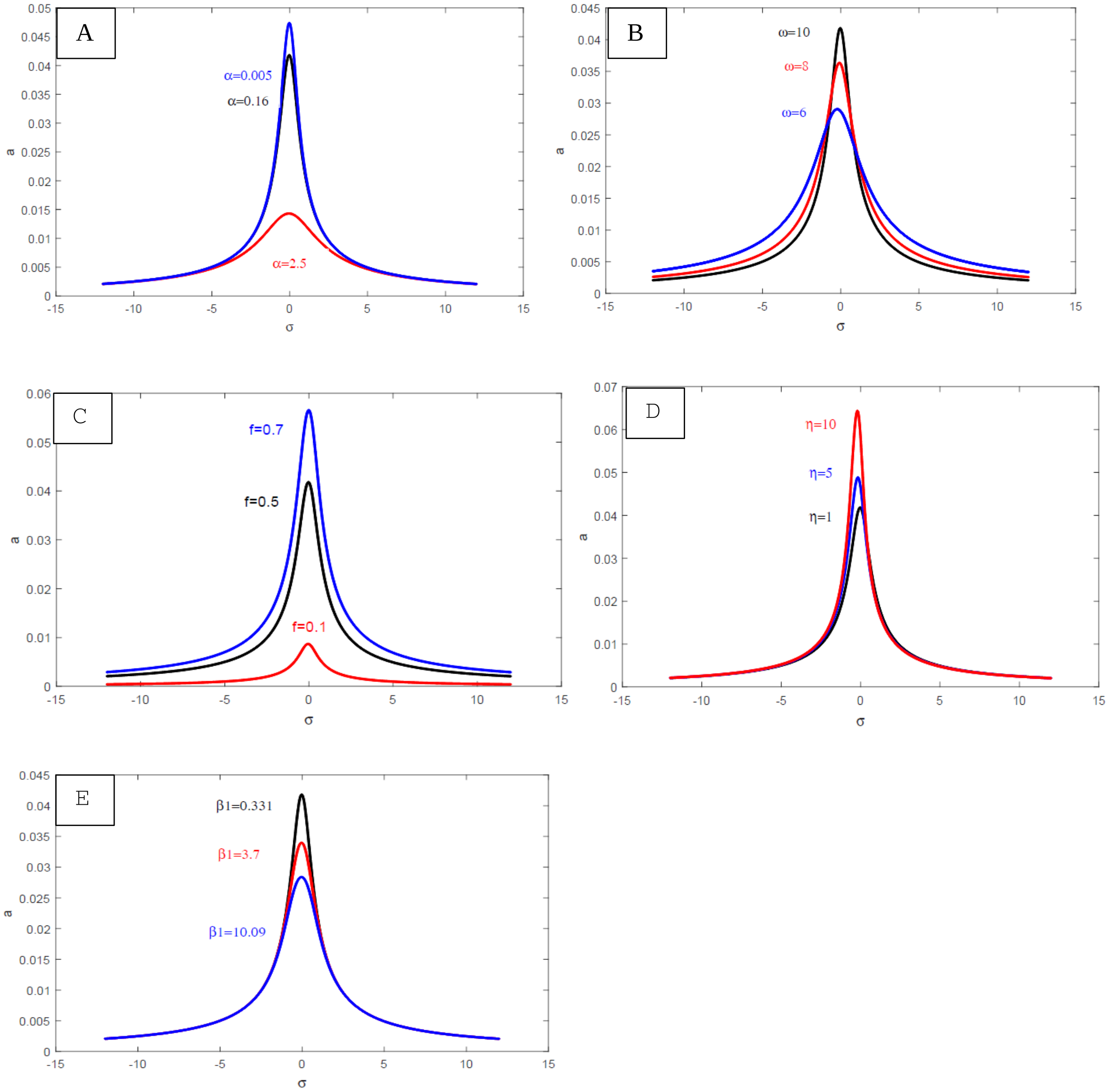

Figure5: the effect of some parameters on frequency-response curve.

The effect of some parameters

Figure6 demonstrate the amplitude of the main system is monotonic decreasing in the linear damping coefficient α and nonlinear parameters $\beta_1, \beta_2 \text{and} \delta_1$ .the amplitude of the system damped like control gain J increased as shown

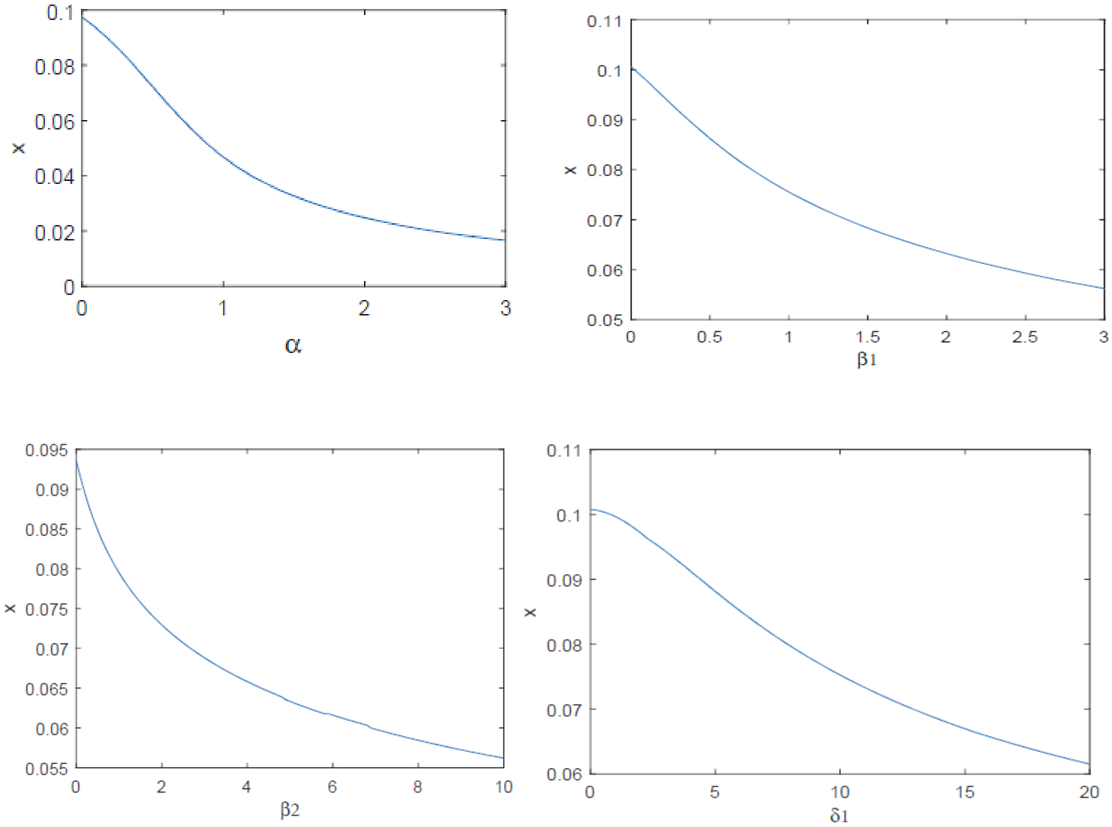

Figure6: the influence of the parameters of the main system without control.

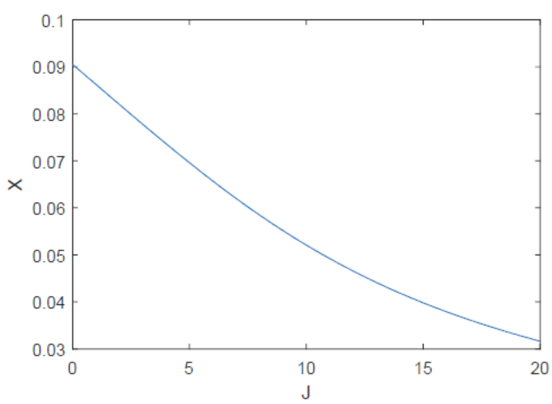

Figure7: the effect integrator gain on the system.

IV.CONCLUSIONS

Within this work, an integral resonate controller has been studied for $\Omega \cong \omega$ of a dynamical system. The method of multiple scales is applied to derive two first order differential equations of the amplitude and phase of the response. The stability and effects of different parameters are studied numerically, the amplitude of the vibrating system was repressed from about 0.1 to about 0.04 and the vibrations were reduced by about 60% from its value without control and the effectiveness of the integral resonant controller E_a is nearly about 2.5. The following remarks can be concluded:

- The increasing of external force f was failing the uncontrolled system.
- The steady state amplitude diminish due to the increasing damping term α.
- The amplitude is a decreasing function of the nonlinear parameters $\beta_1, \beta_2 \text{and} \delta_1$.
- The response of the controlled system decreased with decreasing the natural frequency ω.

REFERENCES

[1] Atkinson, K., Han, W., &Stewart, D.E.: Numerical solution of ordinary differential equations, John Wiley& sons. (2011).

[2] AmerY.A, El-sayedA.T, Abd El-salam, M.N.: Outcomes of the NIPPF controller linked to a hybrid Rayleigh-Van der Pol-Duffing Oscillator. Journal of Control Engineering and Applied Informatics 22(3):33-41. (2020).

[3] Amer,Y.A., El-sayed,A.T, Abd El-salam, M.N :A Suitable active control for suppression the vibrations of a cantilever beam. (2022).

[4] Bauomy, H.S. & EL-Sayed A. T: Mixed controller (IRC+NSC) involved in the harmonic vibration response cantilever beam model. Measurement and Control 53(9-10): 1954-1967. (2020).

[5] Boru, F. E.: Numerical and experimental response and stability investigations of anisotropic rotor-bearing systems, Kassel University press GmbH. (2010).

[6] Eftekhari, M., Ziaei-Rad, S., Mahzoon, M.: Vibration suppression of a symmetrically cantilever composite beam using internal resonance under chord wise base excitation. Int. J. Non-Linear Mech. **48**, 86–100. (2013).

[7] Kandi, A., El-Gohary, H. A.: Suppressing the nonlinear vibrations of a compressor blade via a nonlinear saturation controller. Journal of Vibration and Control24 (8): 1488-1504. (2018).

[8] Li, X., Ji, J.C., Hansen, C.H., Tan, C.:The response of a Duffing–van der Pol oscillator under delayed feedback control. J. Sound Vibr. 291 (3), 644–655. (2006).

[9] MACCARi, A.: Vibration control for the primary resonance of a cantilever beam by a time delay state feedback. Journal of Sound and Vibration 259(2): 241-251. (2003).

[10] Nayfeh, A. H. and Mook, D. t.: Nonlinear oscillations, John Wiley &Sons. (2008).

[11] Omidi, E. and Mahmoodi. S. N.: Sensitivity analysis of the nonlinear integral positive position feedback and integral resonant controllers on vibration suppression of nonlinear oscillatory systems." Communications_in Nonlinear Science and Numerical Simulation **22**(1-3): 149-166. . (2015).

[12] Sadri, M. and Younesian. D.: Nonlinear harmonic vibration analysis of a plate-cavity system. Nonlinear Dynamics **74**: 1267-1279. (2013).

[13] Saeed, N. A., Awwad,M.E, El-Meligy,MA and Naser,EA.:Sensitivity analysis and vibration control of asymmetric nonlinear rotating shaft system utilizing 4-pole AMBs as an actuator. European Journal of Mechanics - A/Solids **86**: 104145. (2021).

[14] Xu, J. and Chung. K.: Effects of time delayed position feedback on a van der Pol–Duffing oscillator. Physica D: Nonlinear Phenomena **180**(1-2): 17-39. (2003).

[15] Yaman, M. and Sen ,S.:Vibration control of a cantilever beam of varying orientation. International Journal of Solids and Structures **44**(3-4): 1210-1220. (2007).