Bulletin of Faculty of Science, Zagazig University (BFSZU)
e-ISSN: 1110-1555
Volume-2024, Issue-1, pp-59-64
https://bfszu.journals.ekb.eg/journal
Research Paper DOI: 10.21608/bfszu.2023.227502.1292

==

Examining the Effect of Kappa Index on Solitary Wave Potential at the plasma environment of Mars

Hala. E. Elgohary[1,] *, Omar. F. Farag[1], and Waleed M. Moslem[2,3]

[1]Physics Department, Faculty of Science, Zagazig University, Zagazig 44519, Egypt University.
[2] Physics Department, Faculty of Science, Port Said University, Port Said 42521, Egypt.
[3]Centre for Theoretical Physics, The British University in Egypt (BUE), El-Shorouk City 43, Cairo, Egypt.

Corresponding author: halaelgohary1122@gmail.com

ABSTRACT **: Nonlinear interactions between nonlinearity, dispersion, and other phenomena in a media give rise to solitons, which are self-reinforcing solitary waves that retain shape and velocity as they propagate across the medium. We investigate electrostatic ion-acoustic solitary waves in a homogeneous, un-magnetized, collision-less plasma environment at 200–500 km for a plasma system consists of three positive ions, whereas the electrons present as a super-thermal charge and follow kappa distribution function. The potential for electrostatic ion-acoustic solitary waves to form in the Martian ionosphere due to the movement of different ionic species was established using the one-dimensional Korteweg-de Vries (KdV) equation has been obtained using the reductive perturbation approach. Graphical representations of numerical studies that explain how super-thermal parameter affects nonlinear ion acoustic waves are shown. The kappa index influence on the soliton waves has been studied. Based on our model an increased kappa index can lead to modifications in soliton amplitude and width.**

KEYWORDS: ***solitary waves, kappa distribution, non-Maxwellian electrons, Reductive perturbation technique.***

Date of Submission: 06-8-2023 Date of acceptance: 20-09-2023

I. INTRODUCTION

Mars, positioned as the fourth celestial body in the solar system, is classified as a terrestrial planet. The planet Mars has garnered considerable scientific attention as a result of its potential for supporting life and its resemblance to Earth's previous environmental conditions **[1-3]**. Mars, like Venus, Titan, and Pluto, does not have a global magnetic field. As a result, solar wind plasma interactions differ from those on magnetic planets such as Earth **[4]**. Therefore, solar radiation penetrates Mars to such an extent that it induces a magnetosphere and an ionosphere resulting various sorts of waves and oscillations **[5]**. One of the most important nonlinear phenomena in the recent plasma research are ion acoustic waves. Over the past few decades, there has been a significant focus on conducting thorough experimental and theoretical investigations to examine the propagation of Ion Acoustic Waves (IAWs) in both laboratory and space environments **[6-8]**. The ion-acoustic soliton waves (IASWs) are a significant focus of study in contemporary plasma research, as they emerge from the intricate interplay between nonlinearity and dispersion **[9]**.

In the field of plasma physics, the study of solitary waves has been of great interest due to their unique and intriguing properties **[10]**. Solitary waves are self-reinforcing solitary waves that maintain their shape and velocity as they propagate through a medium, which sets them apart from other types of waves **[11, 12]**. They are nonlinear phenomena that arise from the interplay between nonlinearity, dispersion, and possibly other effects in the medium **[13]**. In recent years, researchers have been investigating the impact of various factors on the characteristics and behavior of solitary waves in plasma environments **[14]**. One particular factor that has gained attention is the kappa index, which characterizes the velocity distribution of particles in a plasma system. The kappa index, denoted as κ, is a measure of the departure of the particle velocity distribution from a

Maxwellian distribution **[15-17]**. It describes the presence of high-energy tail particles in the plasma, which can significantly affect the wave dynamics **[18, 19]**.

The kappa index is linked to the degree of non-thermal or super-thermal particles present in a plasma **[20]**. These particles arise due to various physical mechanisms, such as wave-particle interactions, particle accelerations, or non-equilibrium states Owen et al. The presence of these non-thermal particles can introduce deviations from the assumptions of Maxwellian distributions, leading to a more complex plasma behavior **[21, 22]**. Understanding the effect of the kappa index on solitary wave potentials is crucial for unraveling the underlying physics and predicting the behavior of waves in different plasma environments **[23, 24]**. The kappa index has been found to influence diverse aspects of solitary waves, including their generation, propagation, and stability **[25]**. The nonlinear behavior of plasma waves and their interaction with the plasma medium can be better understood if the effect of the kappa index on solitary wave potentials is studied. Theoretical and numerical studies have been conducted to explore the relationship between the kappa index and solitary wave properties **[26-28]**. These investigations have revealed that an increased kappa index can lead to modifications in the solitary amplitude, width, and stability. Additionally, the presence of non-thermal particles with high-energy tails can have significant implications for the formation and dynamics of solitary waves **[29-31]**.

Understanding the impact of the kappa index on solitary wave potentials has practical implications in various astrophysical and laboratory plasma contexts **[16]**. For instance, it can provide valuable insights into the behavior of solitary waves in space plasmas, such as the ionosphere of planets like Mars, where non-Maxwellian particle distributions are observed **[32]**. In this study, we aim to investigate and analyze the effect of the kappa index on solitary wave potentials in plasma environment of the planet Mars. By using theoretical models, numerical simulations, and possibly experimental data, we will explore how the presence of non-thermal particles with high-energy tails influences the characteristics and behavior of solitary waves. Our findings will contribute to the understanding of nonlinear wave phenomena in plasma and open up possibilities for controlling and manipulating plasma waves through adjustments in the kappa index.

II. BASIC EQUATIONS

Early observations of the day-side Martian ionosphere reveal that the ionosphere of Mars is dominated by three positive ions [33, 34] and non-Maxwellian electrons [4].We utilize the fluid model for the three ions, whereas we used the kappa distribution function for the electron's distribution. The basic equations for the ions are introduced as follows [35]

(i) Normalized continuity equation for the three ions:

$$\partial n_1/\partial t + \partial(n_1 u_1)/\partial x = 0, \quad (1)$$

$$\partial n_2/\partial t + \partial(n_2 u_2)/\partial x = 0, \quad (2)$$

$$\partial n_3/\partial t + \partial(n_3 u_3)/\partial x = 0, \quad (3)$$

(ii) Normalized equation of motion for the three ions:

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} + 3\,\alpha_1\, n_1 \frac{\partial n_1}{\partial x} + \frac{\partial \varphi}{\partial x} = 0, \quad (4)$$

$$\frac{\partial u_2}{\partial t} + u_2 \frac{\partial u_2}{\partial x} + 3\,\alpha_2\, \frac{1}{\sigma_1}\, n_2 \frac{\partial n_2}{\partial x} + \frac{1}{\sigma_1}\frac{\partial \varphi}{\partial x} = 0, \quad (5)$$

$$\frac{\partial u_3}{\partial t} + u_3 \frac{\partial u_3}{\partial x} + 3\,\alpha_3\, \frac{1}{\sigma_2} n_3 \frac{\partial n_3}{\partial x} + \frac{1}{\sigma_2}\frac{\partial \varphi}{\partial x} = 0, \quad (6)$$

Here, n1,2,3, represents the density of three ions, u1,2,3 are the ions velocities, KB is the Boltzmann's constant, α1,2,3 are the relative temperatures of the ions with respect to the electron temperature where α1,2,3 = T1,2,3/ Te. In this model, we assumed that the three ions have the different temperatures. σ1,2 are the relative masses of the second and the third ions where σ1=m2/m1, σ2=m3/m1, where m refer to the molecular mass of the ion. finally, φ is the electrostatic potential.

For the Non-Maxwellian electrons, we use kappa distribution function f (v)κ α (1 + v2/ κ v2κ) −(κ +1), which has frequently been used to understand and evaluate data of these observed particles in astrophysics and space plasmas. Kappa distribution function is is specified by the index κ and may be reduced to the Maxwellian distribution as κ →∞. The effective thermal speed in this case is vk, which is connected to the known thermal speed vth= (2KB T /m)1/2 by the relation vk2=[(κ − 3/2)/κ] vth 2 and κ denote the slope of the energy spectrum of the super thermal particles that make up the tail of the velocity distribution function. The normalized electrons density number distribution based on kappa distribution function is given by:

$$n_e = \left[1 - \frac{\varphi}{\kappa - \frac{3}{2}}\right]^{-\kappa + \frac{1}{2}} \quad (7)$$

equations (**1- 7**) are connected via Poisson's equation, which is given as

$$\frac{\partial^2 \varphi}{\partial x^2} = \gamma_1 n_e - n_1 - \gamma_2 n_2 - \gamma_3 n_3 \quad (8)$$

Where $\gamma_1 = n_{e0}/n_{10}$, $\gamma_2 = n_{20}/n_{10}$ and $\gamma_3 = n_{30}/n_{10}$, where n_{10}, n_{20}, and n_{30} are the unperturbed densities of the ions. Using the reductive perturbation technique (RPT) [36], in which the stretched space-time coordinate is introduced as $\zeta = \varepsilon 1/2\ (x - \lambda t)$ and $\tau = \varepsilon\ 3/2\ t$, we explore the fundamental features of solutions with small but finite amplitudes. Here λ, is the linear phase velocity normalized by the ion acoustic speed Cs, and ε is a small parameter that measures the strength of the nonlinearity. The dependent variables ni, uj, and φ can be stretched in power series of ε about their equilibrium as:

$$n_{1,2,3} = 1 + \varepsilon\, n^{(1)}{}_{1,2,3} + \varepsilon^2 n^{(2)}{}_{1,2,3} + \ldots, \quad (9)$$

$$u_{1,2,3} = \varepsilon\, u^{(1)}{}_{1,2,3} + \varepsilon^2 u^{(2)}{}_{1,2,3} + \ldots, \quad (10)$$

$$\varphi = \varepsilon\, \varphi^{(1)} + \varepsilon^2 \varphi^{(2)} + \ldots.. \quad (11)$$

Using the stretching space-time coordinates with the aid of the expansions (9)-(11) and isolate the distinct orders in ε, then using some algebraic methods, we obtain a Korteweg-de Vries (KdV) type nonlinear partial differential equation

$$\partial\varphi^{(1)}/\partial\tau + M\,N\,\varphi^{(1)}\,\partial\varphi^{(1)}/\partial\zeta + M\,\partial^3\varphi^{(1)}/\partial\zeta^3 = 0 \quad (12)$$

The dispersion and nonlinear factors are represented by coefficients M and N. They are presented as

$$M = \frac{1}{2}\left[\frac{\lambda}{(\lambda^2 - 3\alpha_1)^2} + \frac{\lambda\gamma_2}{\sigma_1(\lambda^2 - \frac{3\alpha_2}{\sigma_1})^2} + \frac{\lambda\gamma_3}{\sigma_1(\lambda^2 - \frac{3\alpha_2}{\sigma_1})^2}\right]^{-1}, \quad (13)$$

$$N = \frac{1}{2}\left[-\gamma_1\frac{(\kappa+\frac{1}{2})(\kappa-\frac{1}{2})}{(\kappa-\frac{3}{2})^2} + \frac{(3\lambda^2+3\alpha_1)}{(\lambda^2-3\alpha_1)^3} + \frac{3\gamma_2(\lambda^2\sigma_1+\alpha_2)}{\sigma_1{}^3(\lambda^2-\frac{3\alpha_2}{\sigma_1})^3} + \frac{3\gamma_3(\lambda^2\sigma_2+\alpha_3)}{\sigma_2{}^3(\lambda^2-\frac{3\alpha_3}{\sigma_2})^3}\right]^{-1}, \quad (14)$$

On solving equation (**9**), we obtain the solution of the solitary potential. A localized solitary solution of Eq. **(9)** is provided by:

$$\varphi = \varphi_{max}\mathrm{sech}^2\,(W/\chi), \quad (15)$$

where $\varphi max = (3U/M\,N)$, and $W = \sqrt{(2M/U)}$ denote the maximum amplitude width of the localized pulse, respectively. For simplicity we have denoted $\varphi\ (1) = \varphi$, and χ is the transformed coordinate of a frame moving with constant solitary speed U, i.e., $\chi = \zeta - U\,\tau$.

III. RESULTS AND DISCUSSION

The concepts of ion acoustic solitary waves and the kappa distribution function hold significance in plasma physics, as they are interconnected. Studying the effects of the spectral index κ on the ion acoustic solitary waves can also provide insights into the dynamics and equilibrium conditions of the plasma system. By manipulating the spectral index, researchers can control the generation and propagation of solitary waves, which can have practical applications in areas such as plasma diagnostics and wave energy conversion. The profile of solitary potential versus the spatial coordinate χ is given by Fig. (**1**) and (**2**) for different values of κ. Based on the observations, the temperature ratios $\alpha_{1,2,3}$ are assumed to have the same value for the three ions.

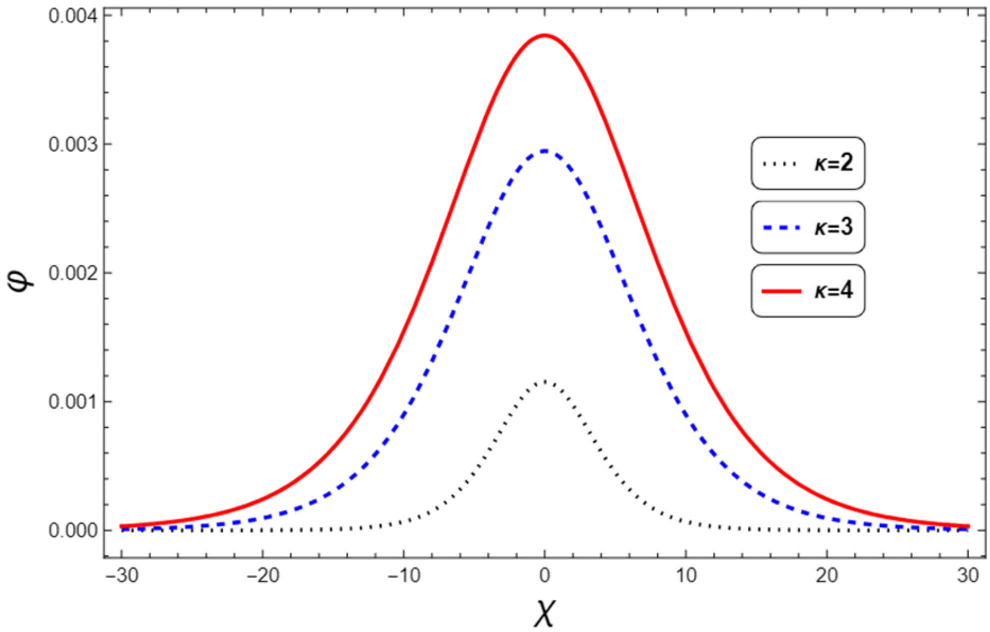

Figure 1: The positive soliton potential profile against x for different values of κ, where U =0.01, α=0.18, σ_1=0.5, σ_2=1.4, γ_1 =0.465, γ_2 =0.4, and γ_3=0.065.

It is clear that the solitary potential amplitude and width increase as κ shifts to higher values. The reason behind this outcome is that the index is linked to the presence of highly energetic electrons in the system. These electrons, having a lower mass than ions, possess higher mobility and can distribute their excess energy more evenly. Consequently, when the parameter is increased, additional energy is introduced into the plasma system. This energy influx causes the amplitude of the waves to increase and acquire a positive potential. It is worth to note that the values of the different parameters such as α, γ1,2,3 are taken from the observations made by Mars Atmosphere and Volatile Evolution (MAVEN) Mission **[4, 34, 37]**.

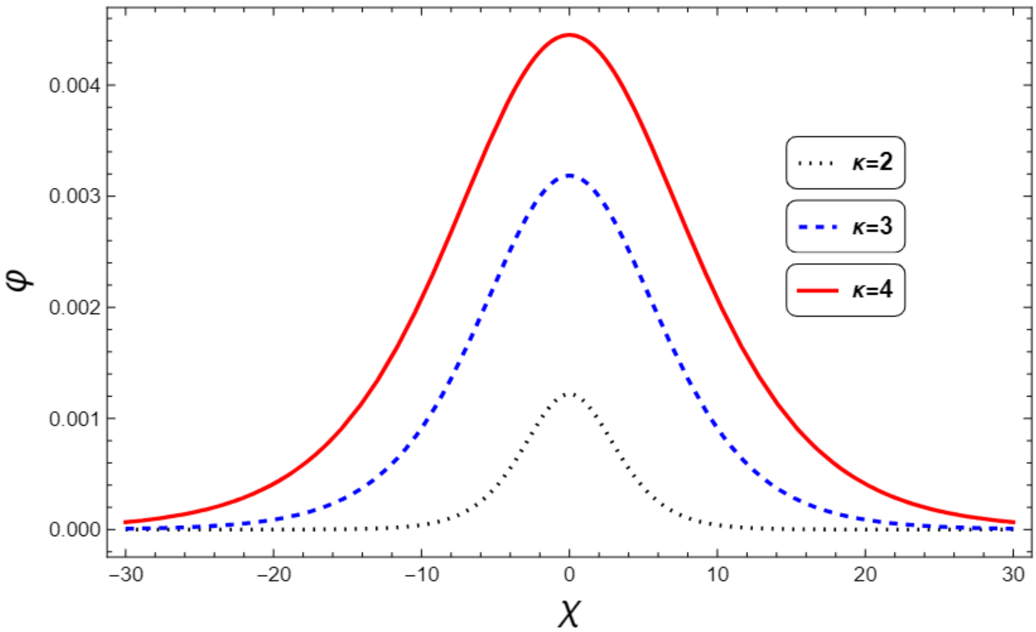

Figure 2: The positive soliton potential profile against x for different values of κ, where U =0.01, α=0.18, σ_1=0.5, σ_2=1.4, γ_1 =0.74, γ_2 =0.64, and γ_3=0.1.

IV. CONCLUION

In summary, the spectral index in the kappa distribution function is a key parameter that influences the properties and behavior of ion acoustic solitary waves in a plasma. The study of these waves and their dependency on the spectral index is important for understanding plasma phenomena and has potential applications in various fields. Overall conclusion, it was demonstrated that as κ increase, the amplitude and width of the solitary pulses' potential will grow.

REFERENCES

1. Dandouras, I., et al., *Future missions related to the determination of the elemental and isotopic composition of earth, moon and the terrestrial planets.* Space science reviews, 2020. **216**: p. 1-49.
2. Dubinin, E., J.G. Luhmann, and J.A. Slavin, *Solar wind and terrestrial planets*, in *Oxford research encyclopedia of planetary science*. 2020.
3. Jakosky, B.M., et al., *The Mars atmosphere and volatile evolution (MAVEN) mission.* Space Science Reviews, 2015. **195**: p. 3-48.
4. Akbari, H., et al., *Spectral analysis of accelerated electron populations at Mars.* Journal of Geophysical Research: Space Physics, 2019. **124**(10): p. 8056-8065.
5. Yadav, V.K., *Plasma waves around Venus and Mars.* IETE Technical Review, 2021. **38**(6): p. 622-661.
6. El-Shafeay, N., et al., *Super rogue waves at Venusian ionosphere and mantle.* Advances in Space Research, 2023.
7. Moslem, W., et al., *Shocklike soliton because of an impinge of protons and electrons solar particles with Venus ionosphere.* Advances in Space Research, 2018. **61**(8): p. 2190-2197.
8. Salem, S., et al., *Ionospheric losses of Venus in the solar wind.* Advances in Space Research, 2020. **65**(1): p. 129-137.
9. Han, J.-N., et al., *Head-on collision of ion-acoustic solitary waves in a weakly relativistic electron–positron–ion plasma.* Physics Letters A, 2008. **372**(27-28): p. 4817-4821.
10. Petviashvili, V.I. and O.A. Pokhotelov, *Solitary Waves in Plasmas and in the Atmosphere*. 1992: Taylor & Francis.
11. Duran, S., *Breaking theory of solitary waves for the Riemann wave equation in fluid dynamics.* International Journal of Modern Physics B, 2021. **35**(09): p. 2150130.
12. Kumar, K., et al., *Reflection of a dust acoustic solitary wave in a dusty plasma.* Physics of Plasmas, 2021. **28**(10).
13. Song, Y., et al., *Recent progress of study on optical solitons in fiber lasers.* Applied Physics Reviews, 2019. **6**(2).
14. Sahoo, H., et al., *Quantum and relativistic effects on the KdV and envelope solitons in ion-plasma waves.* IEEE Transactions on Plasma Science, 2021. **50**(6): p. 1610-1623.
15. Dudík, J., et al., *Nonequilibrium processes in the solar corona, transition region, flares, and solar wind (invited review).* Solar Physics, 2017. **292**: p. 1-72.
16. Livadiotis, G. *Kappa distributions: Thermodynamic origin and Generation in space plasmas*. in *Journal of Physics: Conference Series*. 2018. IOP Publishing.
17. Livadiotis, G., *Kappa distributions: Statistical physics and thermodynamics of space and astrophysical plasmas.* Universe, 2018. **4**(12): p. 144.
18. Bingham, R., J. Mendonca, and P. Shukla, *Plasma based charged-particle accelerators.* Plasma physics and controlled fusion, 2003. **46**(1): p. R1.
19. Shahmansouri, M. and E. Astaraki, *Transverse perturbation on three-dimensional ion acoustic waves in electron–positron–ion plasma with high-energy tail electron and positron distribution.* Journal of Theoretical and Applied Physics, 2014. **8**: p. 189-201.
20. Leubner, M.P., *A nonextensive entropy approach to kappa-distributions.* Astrophysics and space science, 2002. **282**: p. 573-579.
21. Bara, D., M. Djebli, and D. Bennaceur-Doumaz, *Combined effects of electronic trapping and non-thermal electrons on the expansion of laser produced plasma into vacuum.* Laser and Particle Beams, 2014. **32**(3): p. 391-398.
22. Pierrard, V. and M. Lazar, *Kappa distributions: Theory and applications in space plasmas.* Solar physics, 2010. **267**: p. 153-174.

23. Howes, G., *A dynamical model of plasma turbulence in the solar wind.* Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 2015. **373**(2041): p. 20140145.
24. Lembège, B., et al., *Selected problems in collisionless-shock physics.* Space Science Reviews, 2004. **110**: p. 161-226.
25. Sarkar, J., et al., *Formation of solitary structures and envelope solitons in electron acoustic wave in inner magnetosphere plasma with suprathermal ions.* Contributions to Plasma Physics, 2020. **60**(7): p. e201900202.
26. Hassan, M., et al., *Dust–ion-acoustic waves in a κ− nonthermal magnetized collisional dusty plasma with opposite polarity dust.* Results in Physics, 2022. **33**: p. 105106.
27. Kourakis, I., S. Sultana, and M. Hellberg, *Dynamical characteristics of solitary waves, shocks and envelope modes in kappa-distributed non-thermal plasmas: an overview.* Plasma Physics and Controlled Fusion, 2012. **54**(12): p. 124001.
28. Lederer, F., et al., *Discrete solitons in optics.* Physics Reports, 2008. **463**(1-3): p. 1-126.
29. Farooq, M., A. Mushtaq, and J. Qasim, *Dissipative ion acoustic solitary waves in collisional, magneto-rotating, non-thermal electron–positron–ion plasma.* Contributions to Plasma Physics, 2019. **59**(1): p. 122-135.
30. Goswami, J., S. Chandra, and B. Ghosh, *Study of small amplitude ion-acoustic solitary wave structures and amplitude modulation in e–p–i plasma with streaming ions.* Laser and Particle Beams, 2018. **36**(1): p. 136-143.
31. Mehdipoor, M. and M. Asri, *Solitary and periodic waves in magneto–rotating plasmas: effect of the Kappa–Cairns distributed electrons.* Advances in Space Research, 2021. **67**(8): p. 2470-2482.
32. Sayed, F., et al., *Three-dimensional propagation of ion-acoustic waves in the plasma environment of the Venusian ionosphere.* Physica Scripta, 2020. **95**(11): p. 115603.
33. Haider, S.A., K.K. Mahajan, and E. Kallio, *Mars ionosphere: A review of experimental results and modeling studies.* Reviews of Geophysics, 2011. **49**(4).
34. Wu, X.S., et al., *The morphology of the topside Martian ionosphere: Implications on bulk ion flow.* Journal of Geophysical Research: Planets, 2019. **124**(3): p. 734-751.
35. Chen, F.F., *Introduction to plasma physics and controlled fusion*. Vol. 1. 1984: Springer.
36. Washimi, H. and T. Taniuti, *Propagation of ion-acoustic solitary waves of small amplitude.* Physical Review Letters, 1966. **17**(19): p. 996.
37. Sakai, S., et al., *Electron energetics in the Martian dayside ionosphere: Model comparisons with MAVEN data.* Journal of Geophysical Research: Space Physics, 2016. **121**(7): p. 7049-7066.