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Normal Mode Analytic Solution for an Orthotropic Micropolar Thermoelastic Medium

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***ABSTRACT* : The purpose of the present paper is to introduce the analytical solution for an orthotropic micropolar thermoelastic medium using three different theories Dual Phase Lag (DPL), Green & Lindsay (G-L) and Green Naghdi type II (G-N II). The heat equation, equation of motion and micro-rotaton couple stress equation gives a system of partial differential equations. Normal mode analysis method is used to convert the system of partial differential equations into a system of ordinary differential equations to get the analytical solution and expressions for displacements, temperature and stresses. The main conclusion in this paper state that displacements, stress and micropoar theory affected by the mechanical load which agree with the physical observations that any change in temperature will lead to change in physical quantites and shows that the phenomena of infinite speed doesn't exist. Numerical results are graphed, comparisons are carried out in the light of theories (DPL) model, (G-L) theory and (G-N II) theory. Micropolar theory has wide applications in industry applications and geophysics.**

KEYWORDS: Orthotropic micropolar thermoealstic solid, DPL, G-N II, G-L

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I. INTRODUCTION

Thermoelasticity is an extension for lame system for elastic bodies which take in consideration also the thermal effects, in which the classical theory of elasticity and the theory of heat conduction in solid bodies are coupled into one branch. This branch attracted many scientists to build it with their theories. Duhamel [1] was the first study the thermal and mechanical effect on solid, the non classical theory of thermoelasticity was introduced by by Lord and Shulman [2] with one relaxation time and the second one by Green and Lindsay [3] theory which includes two relaxation times and modified all equations of the coupled theory, not only heat equation, Green-Naghdi [4] theory introduce three models of their theories as G-N I, II and III models. The first model is corresponding to the classical thermoelastic model. This model admits undamped thermoelastic waves in thermoelastic material and is best known as the theory of thermoelasticity without energy dissipation. The third model includes the previous two models as special cases, and admits dissipation of energy in general.

In continuum mechanics, the material points which are assumed to be infinitesimal size carry the local physical properties of the body, so it is necessary to identify these particles as microparticles. If they are deformable, this continuum called micromorphic. If they are rigid the continuum is called micropolar. In the classical theory of thermoelasticity which introduced was unable to investigate the deformation properties of solids, so Eringen [5-7] introduce a development to it by the theory of micropolar elasticity. Boschi and Iesan [8] have given detailed reviews on the generalized theory of micropolar thermoelasticity which permits heat transmission as thermal waves of finite speed.

Some differences between the classical theory and the experiments are found in some problems like those which consider stress gradient occur. The stress concentration of holes, cracks, important from the point of view of safety problems in engineering structures. The difference between the classical theory and the experiments is obviously found in dynamical problems, in which elastic vibrations characterized by high frequencies and small wavelengths, are observed when ultrasounds applied. The microparticles has a high impact on the real deformation process, which means the effects of microparticles very important because new types of waves appear which were not in the classical theory.

There are solutions in some problems in the classical theory of elasticity in which the stress tensor can't be symmetry for example the rectangular wedge with tangent load applied on one of its edge. From physical point of view in micropolar continuum, each particle is considered as infinisimal rigid body. The cooserat [9] (or micropolar) continuum theory is one of the most prominent continuum theories where rigid particles has six degrees of freedom three displacements and three rotations, and in addition to original stresses the couple stresses is introduced. Dost and Tabarrok [10] introduced the micropolar generalized thermoelasticity by using Green-Lindsay theory. Chandrasekhariah [11] formulated a theory of micropolar theory which contain heat-flux in the constitutive variables.

2 Basic equations

$$t_{ji,j} = \rho \ddot{u}_i \tag{1}$$

$$m_{ik,j} + \varepsilon_{ijk} t_{ij} = \rho J \ddot{\phi}_k \tag{2}$$

$$k_1^* \frac{\partial^2 T}{\partial x_1^2} + k_2^* \frac{\partial^2 T}{\partial x_2^2} = \left(\tau_1 \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\rho c^* T + \beta_1 T_0 \frac{\partial u_1}{\partial x_1} + \beta_1 T_0 \frac{\partial u_2}{\partial x_2}\right) \tag{3}$$

3 Formulation of the problem

From eq. (1)

$$\rho \ddot{u}_1 = A_{11} \frac{\partial^2 u_1}{\partial x_1^2} + A_{12} \frac{\partial^2 u_2}{\partial x_1 \partial x_2} - \beta_1 (1 + \tau_1 \frac{\partial}{\partial t}) T_{,1} + A_{77} \frac{\partial^2 u_2}{\partial x_1 \partial x_2} + A_{88} \frac{\partial^2 u_1}{\partial x_2^2} + (A_{88} - A_{78}) \frac{\partial \phi_3}{\partial x_2} \tag{4}$$

$$\left(A_{11} \frac{\partial^2}{\partial x_1^2} + A_{88} \frac{\partial^2}{\partial x_2^2}\right) u_1 + (A_{12} + A_{78}) \frac{\partial^2 u_2}{\partial x_1 \partial x_2} - (A_{78} - A_{88}) \frac{\partial \phi_3}{\partial x_2} - \beta_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T_{,1} = \rho \ddot{u}_1 \tag{5}$$

$$\rho \ddot{u}_2 = A_{77} \frac{\partial^2 u_2}{\partial x_1^2} + A_{78} \frac{\partial^2 u_1}{\partial x_1 \partial x_2} - \beta_1 (1 + \tau_1 \frac{\partial}{\partial t}) T_{,2} + A_{12} \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + A_{12} \frac{\partial^2 u_2}{\partial x_2^2} + (A_{78} - A_{77}) \frac{\partial \phi_3}{\partial x_1} \tag{6}$$

$$\left(A_{77} \frac{\partial^2}{\partial x_1^2} + A_{22} \frac{\partial^2}{\partial x_2^2}\right) u_2 + (A_{12} + A_{78}) \frac{\partial^2 u_1}{\partial x_1 \partial x_2} - (A_{78} - A_{77}) \frac{\partial \phi_3}{\partial x_2} - \beta_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T_{,2} = \rho \ddot{u}_2 \tag{7}$$

From eq. (2)

$$\left[B_{66} \frac{\partial^2}{\partial^2 x_1^2} + B_{44} \frac{\partial^2}{\partial^2 x_2^2} - (A_{77} - 2A_{78} + A_{88})\right] \phi_3 + (A_{77} - A_{78}) \frac{\partial u_2}{\partial x_1} + (A_{78} - A_{88}) \frac{\partial u_1}{\partial x_2} = \rho J \ddot{\phi}_3 \tag{8}$$

To put these equations in non-dimensional form

$(x_1, x_2) = \frac{c_1}{w^*}(x_1', x_2')$, $(u_1, u_2) = \frac{\beta_1 T_0}{\rho c_1 w^*}(u_1', u_2')$, $\phi_3 = \frac{\beta_1 T_0}{\rho c_1^2} \phi_3'$, $t_{ij} = \beta_1 T_0 t_{ij}'$, $m_{23} = \frac{c_1 \beta_1 T_0}{w^*} m_{23}'$,

$T = T_0 T'$, $t = \frac{1}{w^*} t'$, $\tau_0 = \frac{1}{w^*} \tau_0'$

By substitute with the non-dimensional form in eqs (5) and (7) and (8)

$$\left(d_1 d_4 \frac{\partial^2}{\partial^2 x_1^2} + \frac{\partial^2}{\partial^2 x_2^2}\right) u_1 + (d_2 + d_3) \frac{\partial^2 u_2}{\partial x_1 \partial x_2} - (d_3 - 1) \frac{\partial \phi_3}{\partial x_2} - d_1 d_4 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T_{,1} = d_1 d_4 \ddot{u}_1 \tag{9}$$

$$\left(\frac{d_2 + d_3}{d_4}\right) \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \left(\frac{\partial^2}{\partial^2 x_2^2} + \frac{d_5}{d_4}\right) u_2 - \left(\frac{d_5 - d_3}{d_4}\right) \frac{\partial \phi_3}{\partial x_1} - \tilde{\beta} d_4 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T_{,2} = d_1 \ddot{u}_2 \tag{10}$$

$$\left[\frac{\partial^2}{\partial^2 x_2^2}+d_1\frac{\partial^2}{\partial^2 x_1^2}-d_7\left(d_5-2d_3+1\right)\right]\phi_3+d_7\left(d_5-d_3\right)\frac{\partial u_2}{\partial x_1}+d_7\left(d_3-1\right)\frac{\partial u_1}{\partial x_2}=d_8\frac{\partial^2\phi_3}{\partial t^2} \quad (11)$$

Eq. of heat (3) after dimensionless

$$\left(\frac{\partial^2}{\partial x_1^2}+\tilde{k}\frac{\partial^2}{\partial x_2^2}\right)T=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\grave{o}\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x_1}+\tilde{\beta}\frac{\partial u_2}{\partial x_2}\right) \quad (12)$$

where $d_1=\frac{A_{11}}{A_{22}}$, $d_4=\frac{A_{22}}{A_{88}}$, $d_2=\frac{A_{12}}{A_{88}}$, $d_3=\frac{A_{78}}{A_{88}}$, $\tilde{\beta}=\frac{\beta_2}{\beta_1}$, $\tilde{k}=\frac{k_2^*}{k_1^*}$, $d_5=\frac{A_{77}}{A_{88}}$, $d_1=\frac{B_{66}}{B_{44}}$,

$d_7=\frac{A_{88}c_1^2}{B_{44}w^{*2}}$, $d_8=\frac{\rho J c_1^2}{B_{44}}$, $\grave{o}=\frac{\beta_1^2 T_0}{\rho w^* k_1^*}$

4 Normal mode analysis

Let [$[u_1,u_2,\phi_3,T]=[u_1^*,u_2^*,\phi_3^*,T^*]e^{wt+ibx}$

Where $\frac{\partial}{\partial t}=w$, $\frac{\partial^2}{\partial t^2}=w^2$, $\frac{\partial}{\partial x_1}=ib$, $\frac{\partial^2}{\partial x_1^2}=-b^2$, $\frac{\partial}{\partial x_2}=$ D, $\frac{\partial^2}{\partial x_2^2}=D^2$

By substitute in eqs (8-12) we get the differential equation

$$[D^8-AD^6+BD^4-CD^2+E](u_1^*,u_2^*,\phi_3^*,T^*)=0 \quad (13)$$

where $A=(A_8A_{14}+A_9\tilde{k}+A_6\tilde{k}+A_{12}+A_3\tilde{k}+A_5\tilde{k})/\tilde{k}$

$$B=(A_8A_{14}A_9+A_8A_1A_{14}+A_9A_6\tilde{k}+A_1A_6\tilde{k}+A_1A_{12}-A_5A_4A_{14}$$
$$+A_5A_2A_9-A_5A_3A_{10}\tilde{k}+A_2A_5A_{12}$$
$$-A_2A_7A_{11}\tilde{k}-A_3A_{11}A_6\tilde{k}-A_3A_{11}A_{12}-A_4A_{13}+A_2A_8A_{13})/\tilde{k}$$

$$C=(A_1A_8A_9A_{14}+A_6A_9A_{12}+A_1\tilde{k}+A_1A_6A_9\tilde{k}+A_1A_9A_{12}+A_1A_6A_{12}-A_7A_{10}+A_4A_5A_{14}$$
$$+A_2A_5A_9A_{12}-$$
$$A_3A_5A_{10}A_{12}+A_{11}A_{14}-A_4A_7A_{11}A_{14}-A_2A_7A_{12}A_{11}-A_3A_6A_{11}A_{12}-A_4A_6A_{13}+A_2A_8A_9A_{13})/\tilde{k}$$

$$E=\left(A_1A_6A_9A_{12}-A_1A_7A_{10}-A_4A_6A_9A_{13}+A_3A_8A_{10}A_{13}\right)/\tilde{k}$$

Eq. (12) can be factored into

$$\{\prod_{n=1}^{4}(\mathrm{D}^2-k_n^2)u_1^*(y),u_2^*(y),\phi_3^*(y),T^*(y)\}=0 \quad (14)$$

Then we have

$$u_1^*=\sum_{n=1}^{4}M_n e^{(-k_n y)}e^{(wt+ibx)},\quad u_2^*=\sum_{n=1}^{4}H_{3n}M_n e^{(-k_n y)}e^{(wt+ibx)},\quad \phi_3^*=\sum_{n=1}^{4}H_{1n}M_n e^{(-k_n y)}e^{(wt+ibx)}$$

$$T^*=\sum_{n=1}^{4}H_{2n}M_n e^{(-k_n y)}e^{(wt+ibx)} \quad (15)$$

where $H_{1n}=\frac{A_{11}k_n-H_{3n}A_{10}}{k_n^2-A_9}$, $H_{2n}=\frac{A_{12}-A_{13}H_{1n}k_n}{k_n^2-A_{14}}$

$$H_{3n}=\frac{A_8k_n^5-\left(A_1A_8+A_4A_5+A_8A_9-A_3A_8A_{11}\right)k_n^3+(A_1A_8A_9+A_4A_5+A_4A_9-A_4A_7A_{11})k_n}{\left(A_2A_8-A_4\right)k_n^4+\left(A_3A_8A_{10}+A_4A_6+A_4A_9-A_8A_2A_9\right)k_n^2-\left(A_7A_4A_{10}+A_4A_6A_9\right)}$$

The non-dimension for tractions and microrotation

$$t_{11}=\frac{\partial u_1}{\partial x_1}+\frac{d_2}{d_1d_4}\frac{\partial u_2}{\partial x_2}-\tilde{\beta}\left(1+\tau_1\frac{\partial}{\partial t}\right)T\ ,\quad t_{12}=\frac{d_5}{d_1d_4}\frac{\partial u_2}{\partial x_1}+\frac{d_3}{d_1d_4}\frac{\partial u_1}{\partial x_2}-\left(\frac{d_5-d_3}{d_1d_4}\right)\phi_3$$

$$t_{21}=\frac{d_3}{d_1d_4}\frac{\partial u_2}{\partial x_1}-\left(\frac{d_3-1}{d_1d_4}\right)\phi_3+\frac{1}{d_1d_4}\frac{\partial u_1}{\partial x_1}\ ,\quad t_{22}=\frac{d_2}{d_4}\frac{\partial u_1}{\partial x_1}+\frac{1}{d_2}\frac{\partial u_2}{\partial x_2}-\tilde{\beta}\left(1+\tau_1\frac{\partial}{\partial t}\right)T$$

$$m_{13}=\frac{d_6}{d_7}\frac{\partial \phi_3}{\partial x_1}\ ,\quad m_{23}=\frac{1}{d_1d_4d_7}\frac{\partial \phi_3}{\partial x_2} \tag{16}$$

By substitute from eqs. (13) into eqs. (14)

$$t_{11}=\sum_{n=1}^{4}H_{5n}M_n e^{(-k_n y)}e^{(wt+ibx)},\ t_{22}=\sum_{n=1}^{4}H_{6n}M_n e^{(-k_n y)}e^{(wt+ibx)}$$

$$t_{12}=\sum_{n=1}^{4}H_{7n}M_n e^{(-k_n y)}e^{(wt+ibx)},\ t_{21}=\sum_{n=1}^{4}H_{8n}M_n e^{(-k_n y)}e^{(wt+ibx)}$$

$$m_{13}=\sum_{n=1}^{4}H_{9n}M_n e^{(-k_n y)}e^{(wt+ibx)},\ m_{23}=\sum_{n=1}^{4}H_{10n}M_n e^{(-k_n y)}e^{(wt+ibx)} \tag{17}$$

Where

$$H_{5n}=ib-\frac{d_2}{d_1d_4}k_nH_{3n}-\tilde{\beta}\left(1+\tau_1 w\right)H_{2n}\ ,$$

$$H_{6n}=i\frac{d_2}{d_1d_4}b-\frac{k_nH_{3n}}{d_1}-\tilde{\beta}\left(1+\tau_1 w\right)H_{2n}$$

$$H_{7n}=i\frac{d_5}{d_4d_1}H_{3n}b-\frac{k_nd_8}{d_1d_4}-\left(\frac{d_5-d_3}{d_1d_4}\right)H_{1n}\ ,\quad H_{8n}=i\frac{d_3}{d_4d_1}H_{3n}b-\frac{k_n}{d_1d_4}-\left(\frac{d_3-1}{d_1d_4}\right)H_{1n}$$

$$H_{9n}=i\frac{d_6}{d_1d_4d_7}bH_{1n}\ ,\quad H_{10n}=\frac{-k_nH_{1n}}{d_1d_4d_7}$$

5 Special cases of thermoelastic theory

The above basic equations are studied for the following theories

Theory	τ_1	τ_0
Green-Lindsay theory (G-L)	0	> 0
Green Naghdi type II (G-N II)	0	1
Chandrasekariah-Tzou theory (DPL)	> 0	> 0

6 Numerical results and discussion

For purpose of numerical the orthotropic micropolar thermoelastic solid, the aluminum has been choose. All the variables are in non-dimensional form

$$d_1=1.02\ ,\ d_2=0.7888\ ,\ d_3=1.9828\ ,\ d_4=6.0224\ ,\ d_5=1.32\ ,\ d_6=1.53\ ,\ d_7=0.00104\ ,\ d_8=1.6543$$

The numerical outlined obtained above was used of the real part of the temperature T, the displacement components $u_1, u_2$and traction components t_{12}, t_{21}, t_{22}. The computations was carried out at $x_1 = 0.8$ over the interval (0, 3).

Figures (1- 9) shows the predict curves using G-L, G-N II and DPL theories. The solid line represents G-L, the dashed is G-N II and the dot line is DPL.

Figure 1 show the distribution of horizontal displacement u_1. It start from positive then decrease to negative values and rises again to positive values and finally converges to zero.

Figure 2 display the distribution of vertical displacement u_2. G-L and G-N II begin with positive values and increase up to $x_2 \simeq 0.5$ then decrease and converges to zero. The DPL theory begin with negative values and increase to positive values up to $x_2 \simeq 0.5$, after that decrease and converges to zero.

Figure 4 show the distribution of temperature T. It's initial value is zero and increase in a very small interval and decease to $x_2 \simeq 0.5$ then increase and finally converges to zero.

Figure 3 introduce the distribution of nicrorotation vector $\emptyset_3$ It start by negative values and increase till $x_2 \simeq 1$, then converges to zero.

Figure 5 display the component of traction t_{12}. The curves start with negative values and rises up to $x_2 \simeq 0.5$, decrease and converges after that to zero.

Figure 6 display the component of traction t_{21}. The curves start with negative values and rises up to $x_2 \simeq 0.5$, decrease and converges after that to zero.

Figure 7 show the component of traction t_{22}. The curves initial values are positive and decrease up to $x_2 \simeq 0.5$, increase and converges after that to zero.

Figure 8 represent the distribution of tangential couple stress m_{13} . The graph begin with negative values followed by increased till x_2 become 0.5 then decrease and finally converges to zero.

Figure 9 describe the distribution of tangential couple stress m_{23} . The graph initial value is zero, increase till x_2 approach 0.5 from left then decrease and converges to zero

Conclusion

The Normal mode analysis technique was used to derive the expressions for stress and temperature distributions due to mechanical and thermal loads. The curves of the traction components t_{21}, t_{12} behave in the same manner increase sharply in the converse of the traction component t_{22} decrease sharply. The couple stress components m_{13}, m_{23} behaves in the same manner. The curves of theories (G-L) and (DPL) are close to each other in most graphs while the curve of (G-N) theory differ from them. For more values of time these distributions values become high, the phenomenon of finite speeds propagation clearly appeared in all these figures.

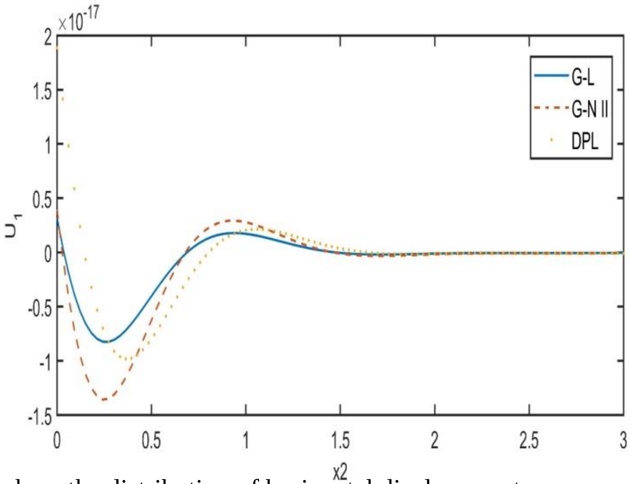

Figure 1 show the distribution of horizontal displacement u_1

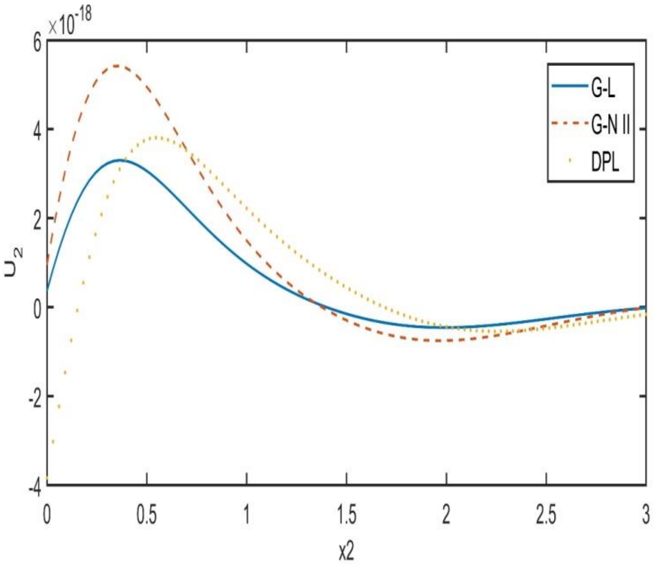

Figure 2 display the distribution of vertical displacement u_2

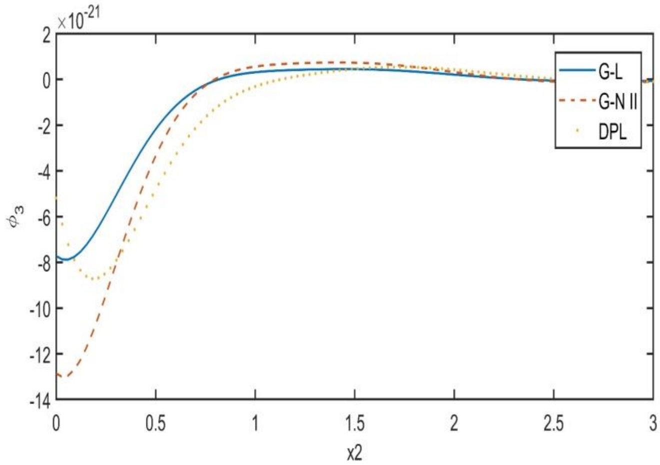

Figure 3 introduce the distribution of microrotation vector $\emptyset_3$

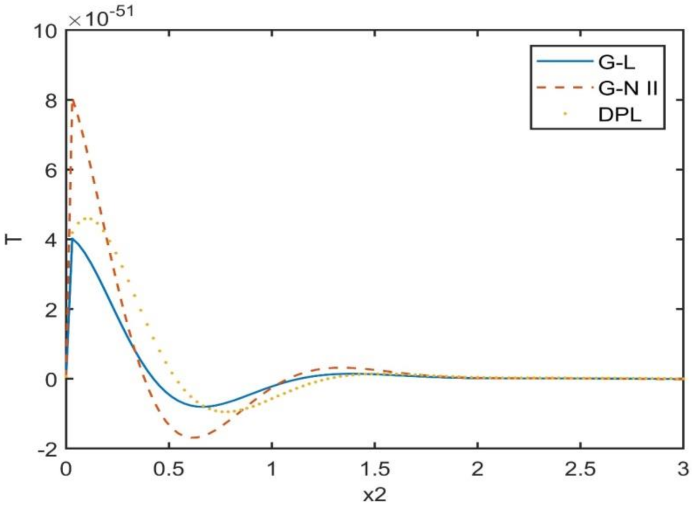

Figure 4 show the distribution of temperature T

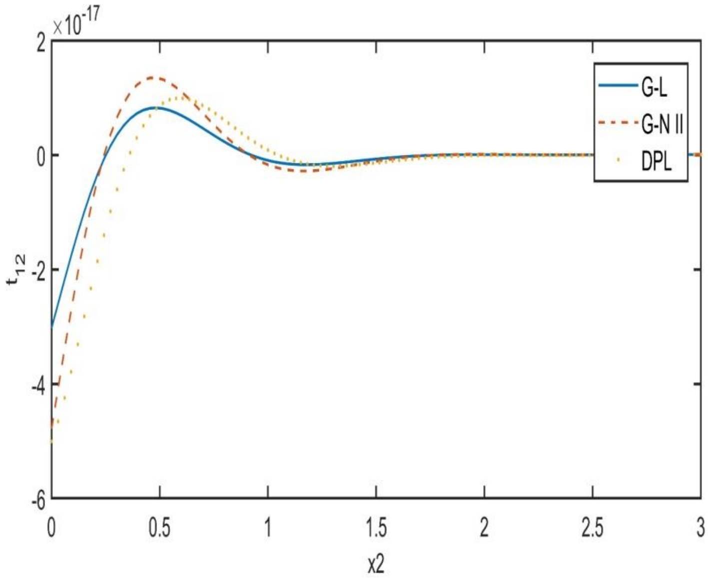

Figure 5 display the component of traction t_{12}

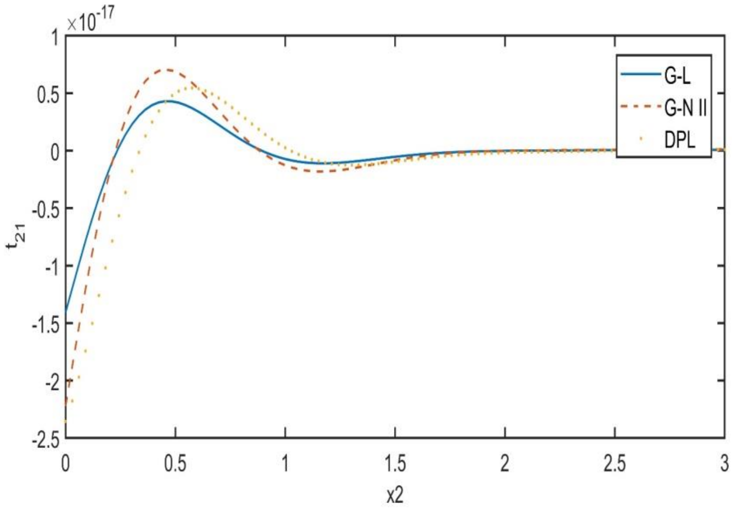

Figure 6 display the component of traction t_{21}

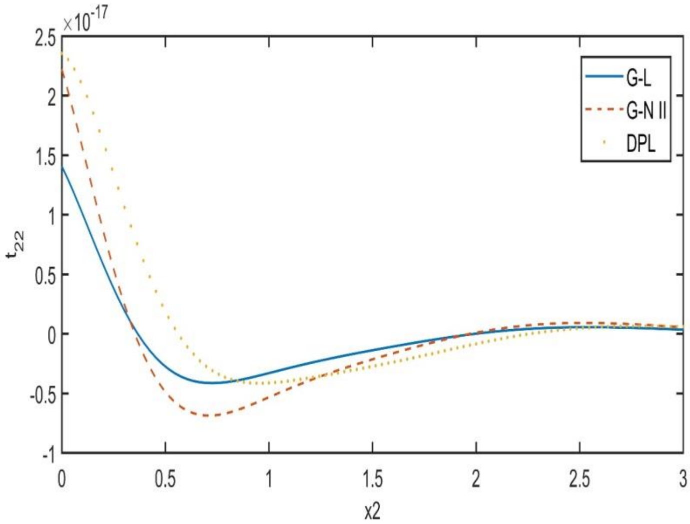

Figure 7 show the component of traction t_{22}

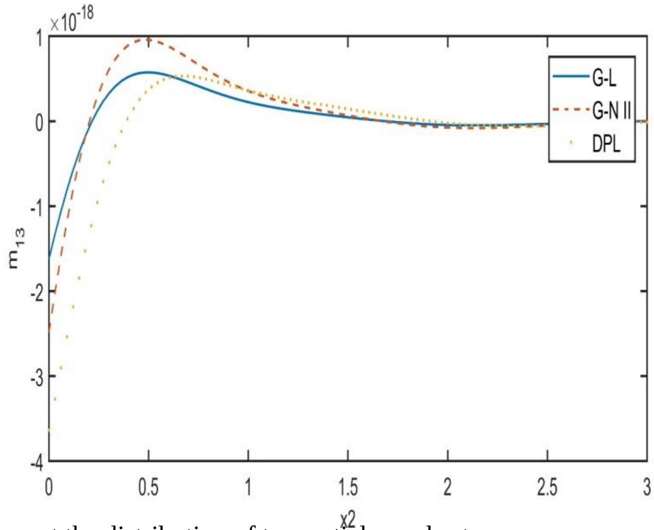

Figure 8 represent the distribution of tangential couple stress m_{13}

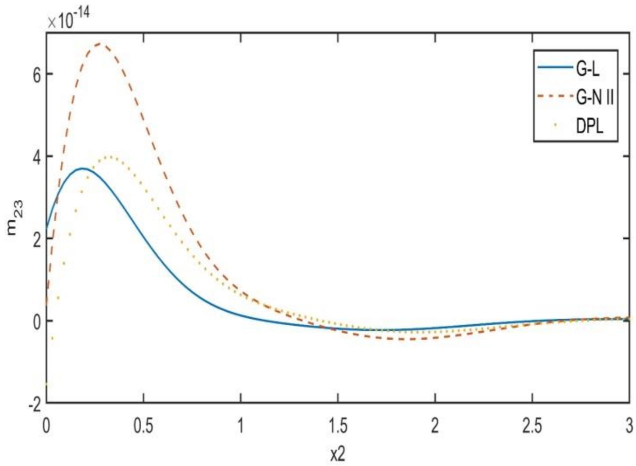

Figure 9 describe the distribution of tangential couple stress m_{23}

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