# The effect of rotation on a nonlocal poro-thermoelastic solid using thermoelasiticity of Choudhuri (3PHL) 

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#### Abstract

In the present study, we discussed the effect of rotation on a nonlocal poro-thermoelastic solid halfspace. A two-dimensional problem is applied using the generalization called the three-phase-lag thermoelasticity theory which is according to Choudhuri model (3PHL). The thermoelasiticity of Choudhuri mdel (3PHL) three-phase-lag model is very useful in the problems of nuclear boiling, exothermic catalytic reactions, phonon-electron interactions, and phonon-scattering. The non-dimensional coupled governing equations of motion are solved to get analytical expressions of the displacement components, thermodynamics temperature, the change in volume fraction field, and the stress components with the aid of normal mode analysis. The numerical results are displayed and presented graphically to depict the effect of the rotation and vertical distance on wave propagation in a nonlocal poro-thermoelastic solid half-space. The results show a valuable contribution to the problem of practical design of such structures, to design stiffness, damping, and so on into the right place of the structure by selecting the appropriate material properties.


KEYWORDS: Rotation, nonlocal parameter, poro-thermoelastic solid, three-phase-lag model, normal mode analysis.

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## I. INTRODUCTION

T The materials with voids are used in a wide range of applications, including catalysis, chemical separations, and tissue engineering. The most important generalization of the classical theory of elasticity is known as the theory of linear elastic materials with voids, this theory is concerned with the materials that have a small distribution and the volumes of voids are included among the kinematics variables and investigate different types of geological and biological materials since the classical theory of elasticity is not sufficient. This theory reduced to the classical theory when the volumes of voids tend to zero. Nunziato and Cowin (1979) studied a non-linear theory of elastic materials with voids. The theory of linear elastic materials with voids was developed by Cowin and Nunziato (1983) to study in a mathematical model the mechanical behavior of porous solids. The behavior of plane waves in the linear elastic material with voids was studied by Puri and Cowin (1985). A theory of thermoelastic materials with voids and without energy dissipation is discussed by Iesan (1986). Cicco and Diaco (2002) discussed a theory of thermoelastic materials with voids in the context of Green-Naghdi without energy dissipation. Abd-Elaziz et. al. (2019) explained the effect of Thomson and initial stress in a pourous thermoelastic media under Green-Naghdi theory. Othman and Abd-Elaziz (2015) discussed the effect of thermal loading due to laser pulse on a pourous thermoelastic media in the context of the dual-phase-lag model. Marin and Öchsner (2017) displayied the effect of a dipolar structure in the context GreenNaghdi thermoelasticity theory.
The angular velocity of the earth, the moon, and other planets have made some problems in poro-thermoelastic rotating media. The thermoelastic rotating media have some problems, which are due to Schoenberg and Censor (1973), Puri (1976), Othman (2010), Abd-Elaziz et al. (2022). In 1972, the theory of nonlocal continuum mechanics was proposed by Eringen. The nonlocal elasticity theory has been established that provides significance for the small-scale influences, also nonlocal strain gradient theory, strain gradient theory, surface
elasticity, and modified couple stress theory. The theory of nonlocal elasticity has been studied by many researchers as Eringen and Edelen (1972), Eringen (1983), Inan and Eringen (1991), Wang and Dhaliwal (1993), Civalek and Demir (2011), Zenkour and Abouelregal (2016), Zhu et al. (2017), and Sarkar et al. (2020)

The aim of this present study is to determine the distributions of the displacement components, the stresses, the change in the volume fraction field, and the temperature in a nonlocal poro-thermoelastic solid under influence of the rotation. A novel model study is illustrated in the context of the three-phase lag model (3PHL). The non-dimensional coupled governing equations are solved using the normal mode analysis. Comparisons are for the field quantities for different values of the rotation.

## II. THE DISCUSSION OF THE PROBLEM

The two-dimensional problem of a nonlocal poro-thermoelastic media under the effect of rotation. We are interested in XY-plane and our dynamic displacement is given as $\boldsymbol{u}=(u, v, 0), \quad w=0, \frac{\partial}{\partial z}=0$.
The constitutive equations can be written by Hetnarski and Eslami (2009), Eringen et al. (1972, 1983, 1991), and Abd-Elaziz et al. (2019):
$\left(1-\varepsilon^{2} \nabla^{2}\right) \sigma_{i j}=\lambda e_{k k} \delta_{i j}+2 \mu e_{i j}+b \varphi \delta_{i j}-\gamma \theta \delta_{i j}$,
where, ${ }^{\varepsilon}=a_{0} e_{0}$ is the elastic nonlocal parameter having a dimension of length, $a_{0}$ is an internal characteristic length and $e_{0}$ a material constant, $\sigma_{i j}$ are the components of stress, $e_{i j}$ are the components of strain, $e_{k k}$ is the dilatation, $\lambda, \mu$ are elastic constants, $\alpha_{t}$ is the thermal expansion coefficient, $\varphi$ is the change in volume fraction field of voids, $\theta=T-T_{0}, T_{0}$ is the reference temperature and $\delta_{i j}$ is the Kronecker's delta.
The equation of motion as Schoenberg and Censor (1973)
$\rho\left\{u_{i, t t}+[\boldsymbol{\Omega} \times(\boldsymbol{\Omega} \times \boldsymbol{u})]_{i}+2\left(\boldsymbol{\Omega} \times \boldsymbol{u}_{, t}\right)_{i}\right\}=\sigma_{j i, j}$,
$\beta \varphi_{, i i}-b e-\alpha_{1} \varphi-\alpha_{2} \varphi_{, t}+\alpha_{3} \theta=\rho \alpha_{4}\left(1-\varepsilon^{2} \nabla^{2}\right) \varphi_{, t t}$,
where $\beta, b, \alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}$ are the material constants accoroding to the presence of voids. The heat conduction equation as Choudhuri (2007)
$K\left(1+\tau_{\theta} \frac{\partial}{\partial t}\right) \nabla^{2} \theta_{, t}+K^{*}\left(1+\tau_{v} \frac{\partial}{\partial t}\right) \nabla^{2} \theta=\left(1+\tau_{q} \frac{\partial}{\partial t}+\frac{1}{2} \tau_{q}^{2} \frac{\partial^{2}}{\partial t^{2}}\right)\left(\rho C_{E} \theta_{, t t}+\gamma T_{0} e_{, t t}+\alpha_{3} T_{0} \varphi_{, t t}\right)$,
where $K^{*}$ is the coefficient of thermal conductivity, $K$ is the additional material constant, $C_{E}$ is the
specific heat at constant strain, $\tau_{\nu}$ is the phase-lag of thermal displacement gradient, ${ }^{\tau} \theta$ is the phase-lag of temperature gradient and $\tau_{q}$ is the phase-lag of heat flux.
We introduce the non-dimension variables as:
$\left(x^{\prime}, y^{\prime}, \varepsilon^{\prime}, u^{\prime}, v^{\prime}\right)=\frac{1}{l_{0}}(x, y, \varepsilon, u, v), \quad\left(t^{\prime}, \tau_{q}^{\prime}, \tau_{\theta}^{\prime}, \tau_{v}^{\prime}\right)=\frac{c_{0}}{l_{0}}\left(t, \tau_{q}, \tau_{\theta}, \tau_{v}\right), \quad \theta^{\prime}=\frac{\gamma \theta}{(\lambda+2 \mu)}, \quad \sigma_{i j}^{\prime}=\frac{\sigma_{i j}}{\mu}$,
$\Omega^{\prime}=\frac{l_{0}}{c_{0}} \Omega, \quad \varphi^{\prime}=\varphi, \quad l_{0}=\sqrt{\frac{K^{*}}{\rho C_{E} T_{0}}}, \quad c_{0}=\sqrt{\frac{\lambda+2 \mu}{\rho}}$.
Introduce Eq. (5) and Eqs. (1) in Eqs. (2)- (4), we get

$$
\begin{equation*}
\left(1-\varepsilon^{2} \nabla^{2}\right)\left(\frac{\partial^{2} u}{\partial t^{2}}-\Omega^{2} u-2 \Omega v_{, t}\right)=\frac{\partial^{2} u}{\partial x^{2}}+B_{1} \frac{\partial^{2} v}{\partial x \partial y}+B_{2} \frac{\partial^{2} u}{\partial y^{2}}-\frac{\partial \theta}{\partial x}+B_{3} \frac{\partial \varphi}{\partial x} \tag{5}
\end{equation*}
$$

$\left(1-\varepsilon^{2} \nabla^{2}\right)\left(\frac{\partial^{2} v}{\partial t^{2}}-\Omega^{2} v+2 \Omega u_{t}\right)=B_{2} \frac{\partial^{2} v}{\partial x^{2}}+B_{1} \frac{\partial^{2} u}{\partial x \partial y}+\frac{\partial^{2} v}{\partial y^{2}}-\frac{\partial \theta}{\partial y}+B_{3} \frac{\partial \varphi}{\partial y}$,
$B_{4}\left(1+\tau_{\theta} \frac{\partial}{\partial t}\right) \nabla^{2} \theta_{, t}+\left(1+\tau_{v} \frac{\partial}{\partial t}\right) \nabla^{2} \theta=\left(1+\tau_{q} \frac{\partial}{\partial t}+\frac{1}{2} \tau_{q}^{2} \frac{\partial^{2}}{\partial t^{2}}\right)\left(B_{5} \theta_{t t}+B_{6} e_{, t}+B_{7} \varphi_{t t}\right)$,
$\varphi_{, i i}-B_{8} e-B_{9} \varphi-B_{10} \varphi_{, t}+B_{11} \theta=B_{12}\left(1-\varepsilon^{2} \nabla^{2}\right) \varphi_{, t t}$,
where, $\quad B_{1}=\frac{\lambda+\mu}{\rho c_{0}^{2}}, \quad B_{2}=\frac{\mu}{\rho c_{0}^{2}}, \quad B_{3}=\frac{b}{\rho c_{0}^{2}}, \quad B_{4}=\frac{K c_{0}}{K^{*} l_{0}}, \quad B_{5}=\frac{\rho C_{E} c_{0}^{2}}{K^{*}}, \quad B_{6}=\frac{\gamma^{2} T_{0} c_{0}^{2}}{K^{*}(\lambda+2 \mu)}$,

$$
B_{7}=\frac{\alpha_{3} B_{6}}{\gamma}, \quad B_{8}=\frac{b l_{0}^{2}}{\beta}, \quad B_{9}=\frac{\alpha_{1} l_{0}^{2}}{\beta}, \quad B_{10}=\frac{\alpha_{2} l_{0} c_{0}}{\beta}, \quad B_{11}=\frac{\alpha_{3} l_{0}^{2}(\lambda+2 \mu)}{\gamma \beta}, \quad B_{12}=\frac{\rho \alpha_{4} c_{0}^{2}}{\beta} .
$$

## III. THE ANALYTICAL METHOD TO SOLVE THE PROBLEM

The solution of the above physical variable can be expressed in terms of normal mode analysis as:
$\left[u, v, \theta, \varphi, \sigma_{i j}\right](x, y, t)=\left[\bar{u}, \bar{v}, \bar{\theta}, \bar{\varphi}, \bar{\sigma}_{i j}\right](x) \exp (\mathrm{i} a y-m t)$
where $\bar{u}(x)$, etc. is the amplitude of the function $u(x, y, t)$ etc., i is the imaginary unit, $m$ (complex) is the time constant and $a$ is the wave number in the $y-$ direction.
From Eq. (10) in Eqs. (6)-(9), thus we get

$$
\begin{align*}
& \left(M_{1} \mathrm{D}^{2}-M_{2}\right) \bar{u}+\left(M_{3} \mathrm{D}^{2}-\mathrm{i} a B_{1} \mathrm{D}-M_{4}\right) \bar{v}-B_{3} \mathrm{D} \bar{\varphi}+\mathrm{D} \bar{\theta}=0,  \tag{11}\\
& \left(-M_{3} \mathrm{D}^{2}-\mathrm{i} a B_{1} \mathrm{D}+M_{4}\right) \bar{u}+\left(M_{5} \mathrm{D}^{2}+M_{6}\right) \bar{v}-\mathrm{i} a B_{3} \bar{\varphi}+\mathrm{i} a \bar{\theta}=0,  \tag{12}\\
& B_{8} \mathrm{D} \bar{u}+\mathrm{i} a B_{8} \bar{v}-\left(M_{7} \mathrm{D}^{2}-M_{8}\right) \bar{\varphi}-B_{11} \bar{\theta}=0, \tag{13}
\end{align*}
$$

$M_{9} \mathrm{D} \bar{u}+\mathrm{i} a M_{9} \bar{v}+M_{10} \bar{\varphi}-\left(M_{11} \mathrm{D}^{2}-M_{12}\right) \bar{\theta}=0$,
where, $M_{1}=\varepsilon^{2}\left(\Omega^{2}-m^{2}\right)-1, \quad M_{2}=\left(1+\varepsilon^{2} a^{2}\right)\left(\Omega^{2}-m^{2}\right)-B_{2} a^{2}, \quad M_{3}=2 \Omega m \varepsilon^{2}, \quad M_{4}=2 \Omega m\left(1+\varepsilon^{2} a^{2}\right)$,

$$
M_{5}=\varepsilon^{2}\left(\Omega^{2}-m^{2}\right)-B_{2}, \quad M_{6}=\left(1+\varepsilon^{2} a^{2}\right)\left(m^{2}-\Omega^{2}\right)+a^{2}, \quad M_{7}=1+\varepsilon^{2} m^{2} B_{12}, \quad M_{8}=m^{2} B_{12}\left(1+a^{2} \varepsilon^{2}\right)+a^{2}+B_{9}+m B_{10},
$$

$M_{9}=m^{2} B_{6}\left(1+m \tau_{q}+0.5 m^{2} \tau_{q}^{2}\right), \quad M_{10}=\frac{B_{7} M_{9}}{B_{6}}, \quad M_{11}=m B_{4}\left(1+m \tau_{\theta}\right)+1+m \tau_{v}, \quad M_{12}=a^{2} M_{11}+m^{2} B_{5}\left(1+m \tau_{q}+0.5 m^{2} \tau_{q}^{2}\right)$.
Solving Eqs. (11)- (14), the following equation can be obtained:
$\left(\mathrm{D}^{8}-E_{1} \mathrm{D}^{6}+E_{2} \mathrm{D}^{4}-E_{3} \mathrm{D}^{2}+E_{4}\right) \bar{\varphi}(x)=0$.
where, $E_{1}=\frac{L_{1}}{L_{5}}, \quad E_{2}=\frac{L_{2}}{L_{5}}, \quad E_{3}=\frac{L_{3}}{L_{5}}, \quad E_{4}=\frac{L_{4}}{L_{5}}$,

$$
\begin{aligned}
L_{1}= & M_{3}^{2} M_{7} M_{12}+M_{3}^{2} M_{8} M_{11}-M_{5} M_{7} M_{9}-B_{1}^{2} M_{7} M_{11} a^{2}+B_{3} B_{8} M_{5} M_{11}+M_{1} M_{5} M_{7} M_{12} \\
& +M_{1} M_{5} M_{8} M_{11}-M_{1} M_{6} M_{7} M_{11}+M_{2} M_{5} M_{7} M_{11}+2 M_{3} M_{4} M_{7} M_{11}, \\
L_{2}= & B_{11} M_{3}^{2} M_{10}+M_{4}^{2} M_{7} M_{11}+M_{3}^{2} M_{8} M_{12}+B_{8} M_{5} M_{10}-M_{5} M_{8} M_{9}+M_{6} M_{7} M_{9}-B_{1}^{2} M_{7} M_{12} a^{2} \\
& -B_{1}^{2} M_{8} M_{11} a^{2}+B_{3} B_{8} M_{5} M_{12}-B_{3} B_{8} M_{6} M_{11}+B_{3} B_{11} M_{5} M_{9}+B_{11} M_{1} M_{5} M_{10}+M_{1} M_{5} M_{8} M_{12} \\
& -M_{1} M_{6} M_{7} M_{12}-M_{1} M_{6} M_{8} M_{11}+M_{2} M_{5} M_{7} M_{12}+M_{2} M_{5} M_{8} M_{11}-M_{2} M_{6} M_{7} M_{11}+2 M_{3} M_{4} M_{7} M_{12} \\
& +2 M_{3} M_{4} M_{8} M_{11}-2 B_{1} M_{7} M_{9} a^{2}-M_{1} M_{7} M_{9} a^{2}+2 B_{1} B_{3} B_{8} M_{11} a^{2}+B_{3} B_{8} M_{1} M_{11} a^{2}, \\
L_{3}= & M_{4}^{2} M_{7} M_{12}+M_{4}^{2} M_{8} M_{11}-B_{8} M_{6} M_{10}+M_{6} M_{8} M_{9}-B_{1}^{2} M_{8} M_{12} a^{2}-B_{3} B_{8} M_{6} M_{12}-B_{3} B_{11} M_{6} M_{9} \\
& -B_{11} M_{1} M_{6} M_{10}+B_{11} M_{2} M_{5} M_{10}+2 B_{11} M_{3} M_{4} M_{10}-M_{1} M_{6} M_{8} M_{12}+M_{2} M_{5} M_{8} M_{12}-M_{2} M_{6} M_{7} M_{12} \\
& -M_{2} M_{6} M_{8} M_{11}+2 M_{3} M_{4} M_{8} M_{12}+2 B_{1} B_{8} M_{10} a^{2}-2 B_{1} M_{8} M_{9} a^{2}+B_{8} M_{1} M_{10} a^{2}-M_{1} M_{8} M_{9} a^{2} \\
& -M_{2} M_{7} M_{9} a^{2}-B_{1}^{2} B_{11} M_{10} a^{2}+2 B_{1} B_{3} B_{8} M_{12} a^{2}+2 B_{1} B_{3} B_{11} M_{9} a^{2}+B_{3} B_{8} M_{1} M_{12} a^{2} \\
& +B_{3} B_{8} M_{2} M_{11} a^{2}+B_{3} B_{11} M_{1} M_{9} a^{2}, \\
L_{4}= & -M_{2} M_{8} M_{9} a^{2}+M_{4}^{2} M_{8} M_{12}+B_{11} M_{4}^{2} M_{10}-B_{11} M_{2} M_{6} M_{10}-M_{2} M_{6} M_{8} M_{12}+B_{8} M_{2} M_{10} a^{2} \\
& +B_{3} B_{8} M_{2} M_{12} a^{2}+B_{3} B_{11} M_{2} M_{9} a^{2}, \\
L_{5}= & M_{3}^{2} M_{7} M_{11}+M_{1} M_{5} M_{7} M_{11} .
\end{aligned}
$$

The solution of Eq. (15), which is bounded as $x \rightarrow \infty$, is given by
$\bar{\varphi}(x)=\sum_{j=1}^{4} R_{j} \exp \left(-k_{j} x\right)$.
Similarly,
$\bar{\theta}(x)=\sum_{j=1}^{4} G_{1 j} R_{j} \exp \left(-k_{j} x\right)$.
$\bar{u}(x)=\sum_{j=1}^{4} G_{2 j} R_{j} \exp \left(-k_{j} x\right)$.
$\bar{v}(x)=\sum_{j=1}^{4} G_{3 j} R_{j} \exp \left(-k_{j} x\right)$.
Using the above results, we get
$\bar{\sigma}_{x x}(x)=\sum_{j=1}^{4} G_{4 j} R_{j} \exp \left(-k_{j} x\right)$,
$\bar{\sigma}_{x y}(x)=\sum_{j=1}^{4} G_{5 j} R_{j} \exp \left(-k_{j} x\right)$,
where $k_{j}^{2}(j=1,2,3,4)$ are the roots of the characteristic equation: $\mathrm{k}^{8}-E_{1} \mathrm{k}^{6}+E_{2} \mathrm{k}^{4}-E_{3} \mathrm{k}^{2}+E_{4}=0$.
$G_{1 j}=\frac{M_{8} M_{9}-M_{7} M_{9} k_{j}^{2}-B_{8} M_{10}}{B_{8} M_{12}-B_{8} M_{11} k_{j}^{2}+B_{11} M_{9}}$,
$G_{2 j}=\frac{G_{1 j}\left(a^{2} B_{8}-B_{11} M_{5} k_{j}^{2}-B_{11} M_{6}\right)-\left(a^{2} B_{3} B_{8}+M_{5} M_{7} k_{j}^{4}-M_{5} M_{8} k_{j}^{2}+M_{6} M_{7} k_{j}^{2}-M_{6} M_{8}\right)}{\mathrm{i} a B_{8} M_{4}-\mathrm{i} a B_{8} M_{3} k_{j}^{2}-a^{2} B_{1} B_{8} k_{j}+B_{8} M_{5} k_{j}^{3}+B_{8} M_{6} k_{j}}$,
$G_{3 j}=\frac{B_{8} k_{j} G_{2 j}+B_{11} G_{1 j}+M_{7} k_{j}^{2}-M_{8}}{\mathrm{i} a B_{8}}, \quad G_{4 j}=\frac{\mathrm{i} a \lambda G_{3 j}-(\lambda+2 \mu)\left(k_{j} G_{2 j}+G_{1 j}\right)+b}{\mu\left(1+\varepsilon^{2} a^{2}-\varepsilon^{2} k_{j}^{2}\right)}, G_{5 j}=\frac{\mathrm{i} a \mu G_{2 j}-\mu k_{j} G_{3 j}}{\mu\left(1+\varepsilon^{2} a^{2}-\varepsilon^{2} k_{j}^{2}\right)}$.

## IV. The boundary conditions

The constants $R_{1}, R_{2}, R_{3}, R_{4}$ can be founded by taking the boundary conditions on the surface at $x=0$ take the form
a) Thermal boundary condition as:

$$
\begin{equation*}
\frac{\partial \theta}{\partial x}=0 . \tag{22}
\end{equation*}
$$

b) Mechanical boundary condition as:

$$
\begin{equation*}
\sigma_{x x}=-f(y, t), \quad \sigma_{x y}=0 . \tag{23}
\end{equation*}
$$

c) Condition on the change in volume fraction field

$$
\begin{equation*}
\varphi=g(y, t) \tag{24}
\end{equation*}
$$

Where $f(y, t)=f_{0} \exp (m t+i a y), g(y, t)=\varphi_{0} \exp (m t+i a y)$ and $f_{0}, \varphi_{0}$ are constants. Introducing Eqs $(16,17)$ and $(20),(21)$ inn Eqs. $(22)-(24)$, we can obtain the following equations

$$
\begin{equation*}
\sum_{j=1}^{4} R_{j}=\varphi_{0}, \quad \sum_{j=1}^{4} k_{j} G_{1 j} R_{j}=0, \quad \sum_{j=1}^{4} G_{4 j} R_{j}=-f_{0}, \quad \sum_{j=1}^{4} G_{5 j} R_{j}=0 \tag{25}
\end{equation*}
$$

With the help of the inverse of the matrix method, thus we get

$$
\left(\begin{array}{l}
R_{1}  \tag{26}\\
R_{2} \\
R_{3} \\
R_{4}
\end{array}\right)=\left(\begin{array}{cccc}
1 & 1 & 1 & 1 \\
k_{1} G_{11} & k_{2} G_{12} & k_{3} G_{13} & k_{4} G_{14} \\
G_{41} & G_{42} & G_{43} & G_{44} \\
G_{51} & G_{52} & G_{53} & G_{54}
\end{array}\right)^{-1}\left(\begin{array}{c}
\varphi_{0} \\
0 \\
-f_{0} \\
0
\end{array}\right)
$$

## V. NUMERICAL CALCULATIONS AND DISCUSSION

To study the effect of rotation on wave propagation in a porous thermoelastic medium, we now present the following numerical results:

$$
\begin{aligned}
& \lambda=3.9 \times 10^{10} \mathrm{~N} \cdot \mathrm{~m}^{-2}, \quad \mu=7.78 \times 10^{10} \mathrm{~N} \cdot \mathrm{~m}^{-2}, \quad \rho=8954 \mathrm{~kg} \cdot \mathrm{~m}^{-3}, \quad C_{E}=383 \mathrm{~J} \cdot \mathrm{~kg}^{-1} \cdot \mathrm{~K}^{-1}, \quad \alpha_{t}=1.78 \times 10^{-3} \mathrm{~K}^{-1}, \\
& f_{0}=0.005, \quad \tau_{q}=9 \times 10^{-7} \mathrm{~s}, \quad \tau_{v}=6 \times 10^{-7} \mathrm{~s}, \quad \tau_{\theta}=7 \times 10^{-7} \mathrm{~s}, \quad K^{*}=386.6 \mathrm{w} \cdot \mathrm{~m}^{-1} \cdot \mathrm{~K}^{-1} \cdot \mathrm{~s}^{-1}, \quad b=1.6 \times 10^{10} \mathrm{~N} \cdot \mathrm{~m}^{-2}, \\
& \alpha_{1}=1.47 \times 10^{10} \mathrm{~N} \cdot \mathrm{~m}^{-2}, \quad \alpha_{2}=7.78 \times 10^{-10} \mathrm{~N} \cdot \mathrm{~m}^{-2}, \quad m=m_{0}+i \xi, \quad m_{0}=0.4, \quad \xi=-0.2, \quad a=0.3, \quad T_{0}=293 \mathrm{~K}, \\
& K=386 \mathrm{w} \cdot \mathrm{~m}^{-1} \cdot \mathrm{~K}^{-1}, \quad \varphi_{0}=0, \quad \alpha_{3}=2 \times 10^{10} \mathrm{~N} \cdot \mathrm{~m}^{-2}, \quad \alpha_{4}=1.753 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{-2}, \quad \beta=2 \times 10^{10} \mathrm{~N} \cdot \mathrm{~m}^{-2}, \quad y=1.5, \quad \varepsilon=0.7 .
\end{aligned}
$$

Figs. 1-4 are graphed to describe the variations of the displacement components $u, v$, the the change in the volume fraction field $\varphi$ and the stress components $\sigma_{x y}$ for different values of the rotation ( ${ }^{\Omega=0,0,2,0.4}$ ). Fig. 1 represents that the variations of displacement $u$ starts with positive values. The values of $u$ decrease in the range $0 \leq x \leq 14$. The increase of the rotation value cause increasing the values of $u$. Fig. 2 shows the variations of the vertical displacement $v$ with distance ${ }^{x}$. The increase of the rotation value causes decreasing the values of $v$. Fig. 3 exhibits that the distribution of the volume fraction field $\varphi_{\text {starts from a zero value and satisfies the boundary }}$ condition. The values of $\varphi$ starts with increase reaching their maximum value and then decrease. The increase of the rotation value causes decreasing the values of $\varphi$. Fig. 4 displays that the distribution of the volume fraction field ${ }^{x y}$ starts from a zero value and satisfies the boundary condition. The increase of the rotation value causes increasing the values of $\sigma_{x y}$.

Figures 5 and 6 are showing 3D surface curves for the thermal temperature $\theta$ and the stress components $\sigma_{x x}$ to study the nonlocal poro-thermoelastic media under the effect of the rotation in the context of Choudhuri model (3PHL). These figures are very important to study the dependence of these physical quantities on the vertical component of distance.

## VI. CONCLUSION

In this problem, we displayed the effect of rotation on a nonlocal thermoelastic porous solid using Choudhuri model (3PHL). The resulting non-dimensional equations were solved by using the normal mode analysis. We can get:
a) The rotation has played a major role in the physical fields which are pretty clear from Figs 1-4.
b) The vertical distance has a great effect on the physical studies fields which are pretty clear from Figs 5, 6.
c) All physical quantities distributions have converged to zero with increasing distance $x$, and all functions are continuous.
d) Analytical solutions based upon normal mode analysis for the thermoelastic problem in solids have been developed and utilized.
e) The deformation of a body depends on the nature of the applied force.

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Fig. 1 Horizontal displacement distribution $u$ for different values of rotation.


Fig. 2 Vertical displacement distribution $v$ for different values of rotation.


Fig. 3 The change in volume fraction field ${ }^{\varphi}$ for different values of rotation.


Fig. 4 Distribution of stress component ${ }^{\sigma_{x y}}$ for different values of rotation.


Fig. 5 Thermal temperature distribution $\theta$ in the context of three-phase-lag model


Fig. 6 Distribution of stress component ${ }^{\sigma_{x x}}$ in the context of three-phase-lag model

